5/3/2022
Final Review

Requested Questions:
- Create a sign chart for 1st and 2nd derivatives x-3sqroot(x). Identify critical points, cusps, and inflection points. Make a graph.

$y = x - 3\sqrt{x}$ vs. $y = x - \sqrt[3]{x}$.

$$y' = 1 - 3\left(\frac{1}{2}\right)x^{-\frac{1}{2}}$$

$$1 - \left(\frac{3}{2}\right)x^{-\frac{1}{2}} = 0$$
$$1 = \frac{3}{2\sqrt{x}}$$

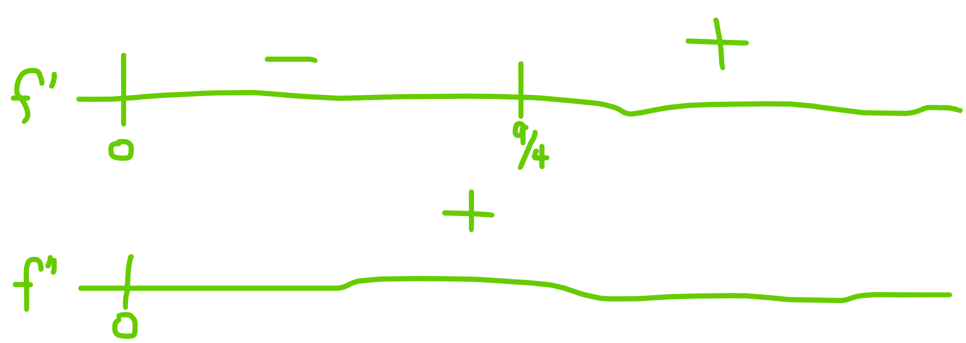
$$2\sqrt{x} = 3$$
$$\sqrt{x} = \frac{3}{2}$$

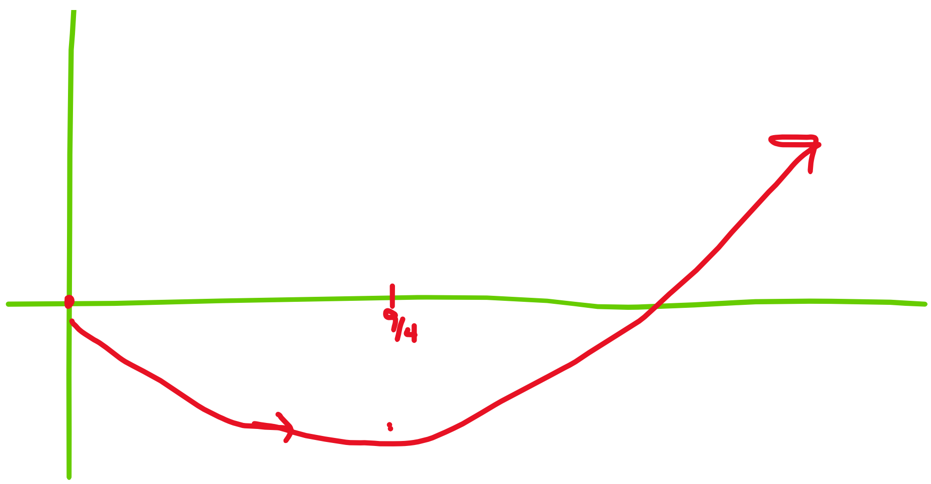
$$x = \frac{9}{4}$$

Undefined when x=0.

$$y'' = -\frac{3}{2}\left(-\frac{1}{2}\right)x^{-\frac{3}{2}} = \frac{3}{4\sqrt{x^3}}$$

If I set this equal to zero, I can't be zero, but it is undefined when x=0.





-- Find antiderivatives. I want to work through the complicated ones -- I am still not sure when I am supposed to be using a formula sheet and when I am supposed to know that I need to substitution or other algebra tricks. I fully understand what it is, but I feel like there are so many variations it's like a 'choose your own adventure' every time.

Change of variable – mostly only for powers of x (short polynomial) times a root (like $\sqrt{x-1}$, or $\sqrt[3]{x+1}$)
For substitution: products of functions where the easier function is related to the derivative of the other. Possibly a function that can't be integrated on its own.
Do algebra to get to basic rules: but you need to know what those are.

- Find the antiderivative and given F(1)=6 find the constant.

$$v(x) = \int \frac{1}{x} - x^2 + 2dx$$

Find the position function, given $s(1) = 6$.

$$\int \frac{1}{x} - x^2 + 2dx = \ln(x) - \frac{1}{3}x^3 + 2x + C$$

$$\ln(1) - \frac{1}{3}(1)^3 + 2(1) + C = 6$$

$$\frac{5}{3} + C = 6$$

$$C = \frac{13}{3}$$

$$s(x) = \ln(x) - \frac{1}{3}x^3 + 2x + \frac{13}{3}$$

- Find derivative of g(x) = 4^t + cos(t) on integral 1, lnx

$$g(x) = \int_1^{\ln x} 4^t + \cos(t)\, dt$$

Plug into x limit (top) to function (ignore constant limits because the derivative will make them zero), but then if the top limit is not just x, multiply by the chain rule.

$$g(x) = \frac{4^t}{\ln 4} + \sin(t)\Big|_1^{\ln x} = \frac{4^{\ln x}}{\ln 4} + \sin(\ln x) - \frac{4^1}{\ln 4} - \sin(1)$$

$$g'(x) = \frac{4^{\ln x}(\ln 4)}{\ln 4}\left(\frac{1}{x}\right) + \cos(\ln x)\left(\frac{1}{x}\right)$$

Using the theorem

$$4^{\ln x}\left(\frac{1}{x}\right) + \cos(\ln x)\left(\frac{1}{x}\right)$$

- Any limit problem that we might encounter on a test.

Any from chapter 2, and L'hopital's

- particle position with maximum

Projectile motion: $h(t) = -\frac{1}{2}gt^2 + v_0 t + h_0$

A ball was shot upward from the surface with an initial velocity of 16 feet per second. When does the ball reach a maximum height, and how high is that?

$$g = 32\frac{ft}{s^2}$$

$$a(t) = -32$$

$$v(t) = \int -32dt = -32t + C$$

$$v(0) = 16$$

$$v(t) = -32t + 16$$

At the maximum, velocity is zero.

$$-32t + 16 = 0$$

$$32t = 16$$

$$t = \frac{1}{2}$$

$$h(t) = \int -32t + 16dt$$

$$h(t) = -16t^2 + 16t + C$$

$$h(0) = 0$$

$$h(t) = -16t^2 + 16t$$

$$h\left(\frac{1}{2}\right) = -16\left(\frac{1}{2}\right)^2 + 16\left(\frac{1}{2}\right) = 4$$

- Force/work problem

Springs and chain problems (no tanks)

Both types: find the force (chain problems=weight times length; for spring problems it's Hooke's Law (F=kx) or it's provided in the questions). The work is the integral of the force. In spring problems you may have to solve for k. Be careful of the units (generally feet or meters). The distance (x values) you integrate over depend on the difference from equilibrium.

For example:
It requires 15 N of force to stretch a spring from its natural length of 10 cm to 15 cm. (5 cm change change).

$$F = kx$$
$$15 = k(0.05)$$
$$k = 300$$

How much work is required to stretch the spring an additional 5 cm.

$$W = \int_{0.05}^{0.10} 300x dx = 150x^2|_{0.05}^{0.10} = 1.125J$$

-- same advice on hyperbolic functions ....what is the sign I should be going to the derivative formula sheet vs. trying to break the problem into parts.