MTH 263, Quiz #2, Spring 2022 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Evaluate each of the following limits. You can use a combination of direct substitution, limit laws, and algebraic simplification techniques to find the limits. (Do not use numerical methods!)

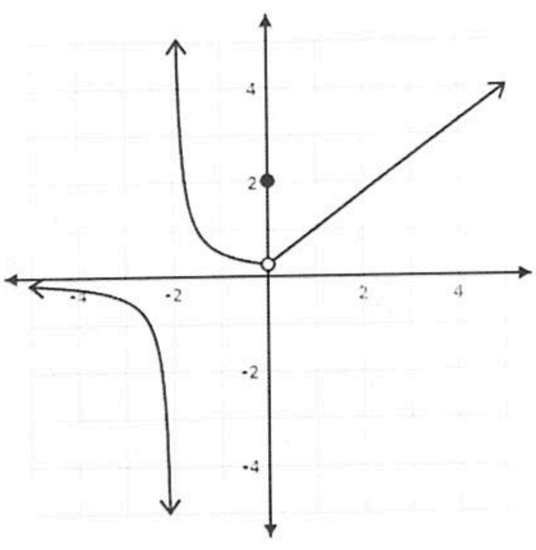
   a. $\lim_{x\to 2} e^{2x-x^2} = e^{2(2)-2^2} = e^{4-4} = e^0 = 1$

   b. $\lim_{x\to 0} \frac{(1+x)^2-1}{x} = \lim_{x\to 0} \frac{\cancel{1}+2x+x^2-\cancel{1}}{x} = \lim_{x\to 0} \frac{\cancel{x}(2+x)}{\cancel{x}} = \lim_{x\to 0} 2+x = 2$

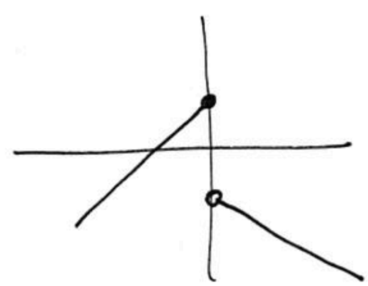
   c. $\lim_{x\to 9} \frac{x-9}{\sqrt{x}-3} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3} = \lim_{x\to 9} \frac{\cancel{(x-9)}(\sqrt{x}+3)}{\cancel{x-9}} = \lim_{x\to 9} \sqrt{x}+3 = \sqrt{9}+3 = 3+3 = 6$

   d. $\lim_{x\to 1} \frac{x^3-1}{x^2-1} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x+1)} = \lim_{x\to 1} \frac{x^2+x+1}{x+1} = \frac{3}{2}$

2. For the function shown in the graph below, identify the location of each discontinuity, and classify it as a) a jump discontinuity, b) an infinite discontinuity, c) a removable discontinuity. Since only two of these can be represented in this graph (since there are only two discontinuities), draw a sketch of a function that contains the missing type.



$x=-2$ infinite discontinuity.
$x=0$ removeable discontinuity



← jump. discontinuity

3. Consider the piecewise function $f(x) = \begin{cases} x^2-3x, & x \le -1 \\ x+b, & x > -1 \end{cases}$. Determine for what value(s) of b is the function continuous?

to be continuous both pieces must have the same value at $x=-1$

$f(-1) = (-1)^2 - 3(-1) = 1+3 = 4$

$x+b = (-1)+b = 4 \rightarrow \boxed{b=5}$

$f(x) = \begin{cases} x^2-3x & x \le -1 \\ x+5 & x > -1 \end{cases}$