MTH 263, Quiz #5, Spring 2022 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Find the indicated derivative of the given function.
   a. $f'(x), f(x) = \sqrt{6 + \sec \pi x^2}$ $= (6 + \sec \pi x^2)^{1/2}$

   $f'(x) = \frac{1}{\cancel{2}}(6 + \sec \pi x^2)^{-1/2}(\sec \pi x^2 \tan \pi x^2)\cancel{2}\pi x$

   $= \boxed{\dfrac{\pi x \sec \pi x^2 \tan \pi x^2}{\sqrt{6 + \sec \pi x^2}}}$

   b. $f'(x), f(x) = \tan(\sec x)$

   $f'(x) = \sec^2(\sec x) \sec x \tan x$

   c. $f''(x), f(x) = (3x - 2)^6$

   $f'(x) = 6(3x-2)^5(3) = 18(3x-2)^5$

   $f''(x) = 18 \cdot 5(3x-2)^4(3) = \boxed{270(3x-2)^4}$

   d. $f'(x), f(x) = \arctan(3x)\,\mathrm{arcsec}(3x)$

   $f'(x) = \frac{3}{1+9x^2}\,\mathrm{arcsec}(3x) + \arctan(3x)\frac{3}{|3x|\sqrt{9x^2-1}}$

   $= \boxed{\dfrac{3\,\mathrm{arcsec}(3x)}{1+9x^2} + \dfrac{\arctan(3x)}{|x|\sqrt{9x^2-1}}}$

2. Find $\frac{dy}{dx}$ for the implicitly defined function $y \sin(xy) = y^2 + 2$.

   $y' \sin xy + y \cos xy \cdot (y + xy') = 2yy'$

   $y' \sin xy + y^2 \cos xy + xyy' \cos xy = 2yy'$

   $y' \sin xy + xyy' \cos xy - 2yy' = -y^2 \cos xy$

   $y'(\sin xy + xy \cos xy - 2y) = -y^2 \cos xy$

   $\boxed{y' = \dfrac{-y^2 \cos xy}{\sin xy + xy\cos xy - 2y}}$

3. Find the equation of the tangent line to the graph $x^2 + 2xy - 3y^2 = 0$ at the point (1,1).

   $2x + 2y + 2xy' - 6yy' = 0$

   $2x + 2y = 6yy' - 2xy'$

   $2x + 2y = y'(6y - 2x)$

   $\boxed{y' = \dfrac{2x + 2y}{6y - 2x}}$

   $y'(1,1) = \frac{2(1) + 2(1)}{6(1) - 2(1)} = \frac{4}{4} = 1$

   $\boxed{y - 1 = 1(x - 1)}$

   $y = x - 1 + 1 \Rightarrow \boxed{y = x}$