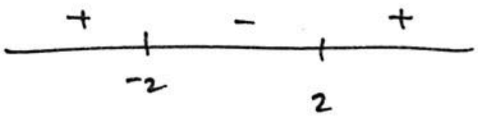
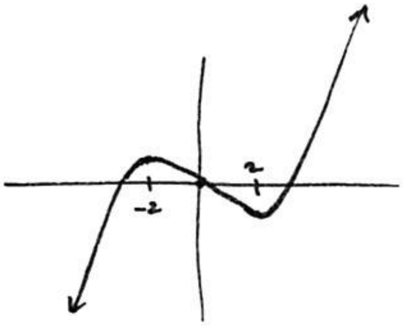
MTH 263, Quiz #9, Spring 2022 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Find all the critical points of the function $f(x) = x^3 - 12x$. Use this information to create a sign chart and sketch the graph.

$f'(x) = 3x^2 - 12 = 3(x^2-4) = 0 \quad x = \pm 2$





2. Find the absolute maximum and absolute minimum of the function $f(x) = x^2 + \frac{2}{x}$ on the interval $[1,4]$.

$f'(x) = 2x - \frac{2}{x^2} = 0 \qquad 2x = \frac{2}{x^2} \to \cancel{2}x^3 = \cancel{2} \to x^3 = 1 \; \boxed{x=1}$

$f(1) = (1)^2 + \frac{2}{1} = 3$ (1,3) absolute minimum

$f(4) = (4)^2 + \frac{2}{4} = 16 + \frac{1}{2} = \frac{33}{2}$ $(4, \frac{33}{2})$ absolute maximum

3. Determine whether the Mean Value Theorem applies to the function $f(x) = x^3 + 2x + 1$ on the interval $[0,6]$. If it does not apply, explain why not. If it does apply, find the point in the interval where the slope of the tangent line is the same as the average rate of change over the entire interval.

the function is continuous on this interval, so yes, it applies

$f(0) = 0^3 + 2(0) + 1 = 1$

$f(6) = 6^3 + 2(6) + 1 = 216 + 12 + 1 = 229$ $\quad \frac{229-1}{6-0} = \frac{228}{6} = 38$

$f'(x) = 3x^2 + 2 = 38$

$3x^2 = 36$

$x^2 = 12$

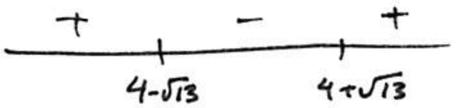
$x = \pm\sqrt{12} = \pm 2\sqrt{3}$

on interval: $2\sqrt{3}$

4. Given $f(x) = x^3 - 4x^2 + x + 2$, find any critical points, any intervals where the graph is increasing and where it is decreasing, find any points of inflection, and intervals where the graph is concave up and concave down.

$f'(x) = 3x^2 - 8x + 1 = 0$

$x = \frac{8 \pm \sqrt{64 - 4(3)(1)}}{2} = \frac{8 \pm \sqrt{52}}{2} = \frac{8 \pm 2\sqrt{13}}{2} = 4 \pm \sqrt{13}$ critical points

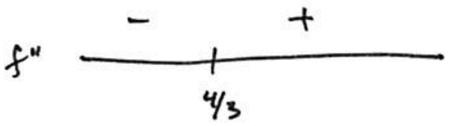


$f''(x) = 6x - 8 = 0$

$6x - 8 = 0$

$6x = 8$

$x = \frac{8}{6} = \frac{4}{3}$ inflection point



increasing $(-\infty, 4-\sqrt{13}) \cup (4+\sqrt{13}, \infty)$

decreasing $(4-\sqrt{13}, 4+\sqrt{13})$

Concave up $(4/3, \infty)$

Concave down $(-\infty, 4/3)$

5. Find any horizontal and vertical asymptotes to the graph $f(x) = \frac{x \sin x}{x^2-1}$. Sketch the graph.

VA : [illegible] $x = \pm 1$

HA: $y = 0$

$$f'(x) = \frac{(x\cos x + \sin x)(x^2-1) - 2x(x\sin x)}{(x^2-1)^2} = \frac{x^3\cos x - x\cos x + x^2\sin x - \sin x - 2x^2\sin x}{(x^2-1)^2}$$

$$= \frac{(x^3-x)\cos x + (-x^2-1)(\sin x)}{(x^2-1)^2}$$

when $x=0$, This is also 0.
critical point

$\pm\pi, \pm 2\pi, \pm 3\pi$, etc are zeros of the function

$(x^3-x)\cos x = (x^2+1)(\sin x)$

$x(x^2-1)\cos x = (x^2+1)(\sin x)$

other critical points appear to be close to $\frac{n\pi}{2}$, but not exactly