MTH 325, Quiz #1, Spring 2023 Name KEY

Instructions: Answer each question as thoroughly as possible. Round answers to 4 decimal places as needed. Exact answers are best when possible. Be sure to answer all parts of each question.

1. Consider the data in the table below of stiffness (x) and thickness (y) of several samples of flame-retardant fabric.

x	7.98	24.52	12.47	6.92	24.11	35.71
y	.28	.65	.32	.27	.81	.57

Find the correlation of the variables (you may use technology to do so).

$r = 0.77287\ldots$

2. Consider the three pairs of observations, $\{(3,9), (5,15), (8,22)\}$. Use then normal equation $B = (A^TA)^{-1}A^TY$ discussed in class to find the coefficients of the linear equation $y = \beta_0 + \beta_1 x$ to model the data. Show your work here. You can use your calculator to test your computations, but you should show all the steps of the solution here by hand. Write your final best-fit equation.

$\beta_0 + 3\beta_1 = 9$
$\beta_0 + 5\beta_1 = 15$
$\beta_0 + 8\beta_1 = 22$

$A = \begin{bmatrix} 1 & 3 \\ 1 & 5 \\ 1 & 8 \end{bmatrix}$, $B = \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$, $Y = \begin{bmatrix} 9 \\ 15 \\ 22 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 8 \end{bmatrix}$ $\quad A^TA = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 8 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 1 & 5 \\ 1 & 8 \end{bmatrix} = \begin{bmatrix} 3 & 16 \\ 16 & 98 \end{bmatrix}$

$A^TY = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 8 \end{bmatrix}\begin{bmatrix} 9 \\ 15 \\ 22 \end{bmatrix} = \begin{bmatrix} 46 \\ 278 \end{bmatrix}$

$A^TAB = A^TY \rightarrow B = \begin{bmatrix} 3 & 16 \\ 16 & 98 \end{bmatrix}^{-1}\begin{bmatrix} 46 \\ 278 \end{bmatrix} = \begin{bmatrix} 1.57894\ldots \\ 2.5789\ldots \end{bmatrix}$

$= \begin{bmatrix} 30/19 \\ 49/19 \end{bmatrix}$

$y = 30/19 + 49/19\,x$

or $y = 1.5789 + 2.5789x$