1/31/2023

Introduction to the course
Set Notation
Venn Diagrams
Logical Notation
Counting

Sets are groups of elements that don't have order.

The sets $\{1,2,3\}$ and $\{1,3,2\}$ are the same set.

Sets are equal when they have all the same elements (in any order).

$$\{b, e, t, s, y\} = \{b, e, s, t, y\} = \{y, t, s, e, b\}$$

The things contained in a set are the elements.

One way to describe a set is to list all the elements inside curly brackets {}.
But another way is to use set notation, set builder notation:

$$\{variable \mid conditions\ on\ the\ variable\}$$

Or

$$\{variable : conditions\ on\ the\ variable\}$$

Example

$$\{x | x \geq 1, x\ is\ real\}$$

Another way to write large set is with ellipses and an established pattern.

$$\{x | x\ is\ an\ integer, x\ is\ even\}$$

$$\{\dots, -4, -2, 0, 2, 4, 6, \dots\}$$

Sets are using given capital variable names: A, B, C,…
Elements in the set are given lower variable names: a,b,c,x,…

$$a \in A$$

"(the element) a is an element of (the set) A" = "a is in A"

$$a \notin A$$

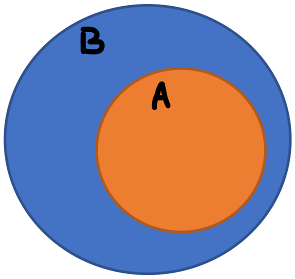
"a is not an element of A"

What if we want to describe the relationship between more than one set?

We have sets A and B.

It's possible that A and B are mutually exclusive (they have no elements in common).
It's possible that one set is entirely contained in the other set.
It's possible that there is some overlap between the sets.

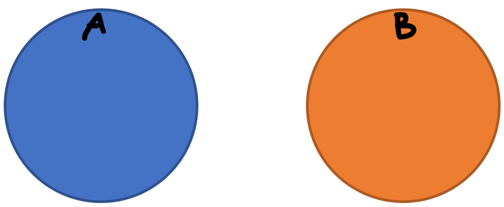
If one set is entirely contained in the other, then we can say $A \subset B$
Read: "A is a subset of B".



A={1,2,3}, B={1,2,3,4,5,6}

This means that everything in A is also in B (but B can have other things in it).

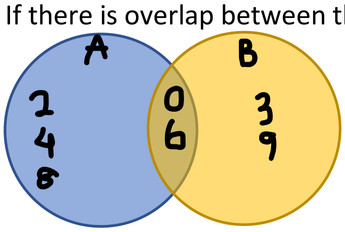
It is incorrect to set $a \subset A$, because a is an element not a set.



mututally exclusive sets have no elements in common.

An example would be $A = \{1,2,3\}, B = \{4,5,6\}$

If there is overlap between the sets, then the Venn diagram will have an overlap in the circles.



The place where the two sets overlap is called the intersection.

A={0,2,4,6,8}, B={0,3,6,9}

The intersection (overlap) of A and B: $A \cap B$
It's what they have in common.

Sets that are mutually exclusive have no intersection. $A \cap B = \emptyset \; or \; \{\}$ (the empty set)

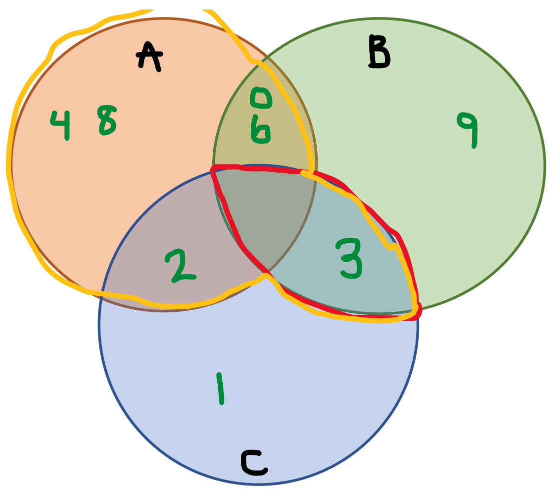
The union of sets A and B = all the elements of A and B together: $A \cup B$={0,2,3,4,6,8,9}

Union of sets: the set of all elements in either set A **or** set B
Intersection of sets: the set of all elements in both set A **and** set B

We can use these same operations/relations with more than two sets, but they can only operate on two sets at a time.

$$A \cup B \cap C = A \cup (B \cap C)$$

If there are no parentheses: intersections go first, then unions.

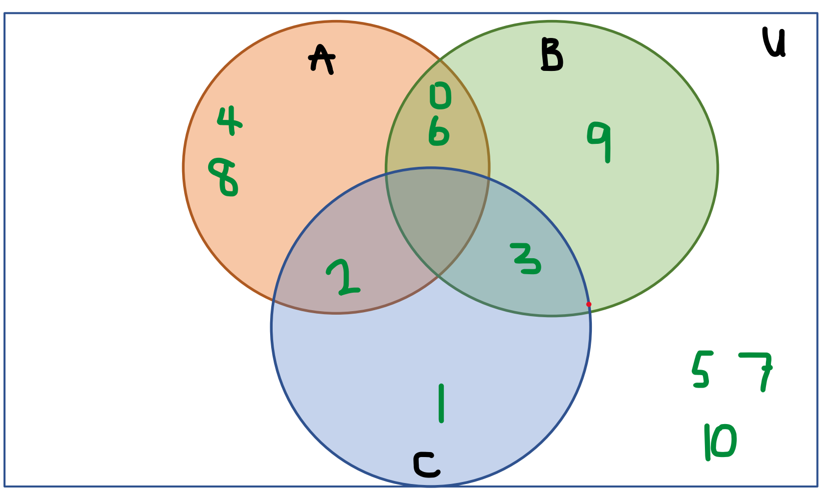


A={0,2,4,6,8},B{0,3,6,9}, C={1,2,3}

$$A \cap B = \{0,6\}, B \cap C = \{3\}, A \cup (B \cap C) = \{0,2,4,6,8,3\}$$

The complement of a set.
The universal set: defined in a particular context, but it is a set for which all other sets (in the given problem) are a subset.

The universal set is U={0,1,2,3,4,5,6,7,8,9,10} (for example).



The complement of set, these are the elements in the universal set that are NOT in the given set.

$$\bar{A}, \sim A, A' \text{: } read\ as \text{ "not A"} = \{1,3,5,7,9,10\}$$

n(A) = the number of elements in A = |A|
AxB is the set of pairs (a,b)

Logical Notation
Logical notation is used to connect statements (sentences).
A statement is a declarative sentence: The sun is shining. The rain is falling. Bobbie is a jerk.
These statements have a truth value. The statement can be true or false.

In logical notation, statements are abstracted to variables, lower case letters, usually starting at p.

The sun is shining = p
The rain is falling = q

The negation of a statement: $\sim p$
The sun is not shining.

~q: The rain is not falling.

$$\sim(\sim q) = q$$

And / or are conjunctions that join to statements:

And: $p \wedge q$ = the sun is shining and the rain is falling. (the intersection ∩ is the set with elements in both A **AND** B.)

Or: $p \vee q$ = the sun is shining or the rain is falling. (the union ∪ is the set with elements in either A **OR** B.)

If, then relation: $p \rightarrow q$: if p, then q: If the sun is shining, then the rain is falling. (read as: p implies q, or if p, then q) (conditional)

In sets, if $A \subset B$, and element of A is also an element of B, being an element of A implies that the element is an element of B; if a is an element of A, then a is also an element of B.

Biconditional is like the equal sign. $p \leftrightarrow q$ p is true if and only if q is true; p and q and both true or false at the same time, their truth values are equal; they are equivalent. (like an equal sign)

$p \leftrightarrow q$: The sun is shining if and only if the rain is falling.

Quantifiers:
∀ = "for all" = this variable is true for the statement in all cases

$$\forall n \text{ in the set of counting numbers}, n > 0$$

The set of all counting numbers is {1,2,3,4…}

∃ = "There exists" = there is at least one value for the variable that makes this true.

$$\exists x \text{ such that } x + 3 = 4$$

∃! = "there exists exactly one (and only one) value for the variable that makes this true.

$$\exists!\, x \text{ such that } x + 3 = 4$$

Vs.

$$\exists x \text{ such that } x^2 = 4$$

This is true because there is "at least one value of x that makes this true", but

$$\exists! \, x \text{ such that } x^2 = 4$$

Is not true, because there isn't only one value that makes it true, there are two: -2, 2

If I used the equation $x^2 = -4$, then all versions of the quantifiers would produce a false statement, because this has no solutions.

Counting
Multiplication Rule
Combinations/Permutations

Multiplication Rule: If I'm selecting multiple things/objects and each thing has a certain number of options, then the total number of possible selections for all the objects together the product of the number of options for each object.

The license plates in a particular state are of the form AAA – 999 where A can be any letter of the alphabet (A-Z), and 9's can be replaced with any single digit number (0-9),

GTY-834

How many possible license combinations are there?

26x26x26x10x10x10=17,576,000

3 pairs of pants, 4 shirts, 2 jackets, and 2 pairs of shoes: 3x4x2x2=48 outfit combinations

Permutations:
Picking from the same set, but you can't repeat a selection
The number of selections available with each pick decreases:
A lottery has 44 balls, and I need to pick 6 balls.
The first ball = 44 options
The second ball = 43 options
Third ball = 42 options
Fourth ball = 41 options…

The total number of outcomes: 44x43x42x41x40x39=5,082,517,440

P = stands for permutation, n = the maximum number of options, r = how many times are we picking from the set.

$$nPr, P(n,r), P_r^n \ldots$$

In this example: $44P6, P(44,6), P_6^{44}$

$$nPr = \frac{n!}{(n-r)!}$$

$$n! = n(n-1)(n-2)\ldots(3)(2)1$$
$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

(one side note: 0!=1)

$$44P6 = \frac{44!}{(44-6)!} = \frac{44!}{38!} = \frac{44 \times 43 \times 42 \times 41 \times 40 \times 39 \times 38 \times 37 \times \ldots \times 2 \times 1}{38 \times 37 \times \ldots \times 2 \times 1}$$

$$= 44 \times 43 \times 42 \times 41 \times 40 \times 39$$

Combinations:
Picking from a fixed set, no repetition and we don't care about the order.
Suppose you work for a company with 25 employees and there is a raffle for a year-end vacation to Aruba. The boss is giving away three vacations.

Combinations are smaller than the permutations for the same number of selections/options, because permutations do care about the order.

$$nCr, C(n,r), \ldots$$
$$nCr = \frac{nPr}{r!} = \frac{n!}{(n-r)!\,r!}$$

$$25C3 = \frac{25!}{(25-3)!\,3!} = \frac{25 \times 24 \times 23 \times 22 \times 21 \times \ldots}{(22 \times 21 \times \ldots)(3 \times 2 \times 1)} = \frac{25 \times 24 \times 23}{3 \times 2} = 25 \times 4 \times 23 = 2300$$