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Simple Interest
Compounding
Amortization Tables
Loans and Savings
Financial Literacy

Simple Interest

$$I = Prt$$

I = interest in dollars
P = principal in dollars (starting amount in the account)
r = interest rate (an annual rate)
t = time (in years)

Suppose you borrow $1000 at 4% simple interest for 2 years. How much interest is paid on the loan? What amount is due back at the end of 2 years?

$$A = P + I = P + Prt = P(1 + rt)$$

I= $80 (see Excel)
A= Amount owed at the end of 2 years = 1000 + 80 = $1,080. (see Excel)

If the problem does not say “compounding” or “compounded” it’s intended to be simple interest.

Suppose you borrow $1000 at 4% simple interest for 9 months. How much interest is paid on the loan? What amount is due back at the end of 9 months?

9/12 = ¾ of a year.

If your problem talked about a certain number of days, divide by 365 to get the equivalent fraction of a year.

One potentially tricky problem might be if the problem gives the interest rate as “per month” or some other time period. Then the time amount needs to be in the same units.

Suppose you borrow $1000 at 1% simple interest per month for 9 months. How much interest is paid on the loan? What amount is due back at the end of 9 months?

Compounding Interest: charges interest on the interest. At the end of a certain period, interest accumulated is added to the principal, and new interest is charged on the new balance.

Compounding interest without payments: setting money aside and letting the interest accumulate.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A= amount in the account after t years
t = time in years
n = the number of times interest is compounded per year
r = annual interest rate
P = principal

Typical values for n:
Annual compounded = 1
Quarterly = 4
Monthly = 12
Bimonthly = 24
Biweekly = 26
Weekly = 52
Daily = 365

Suppose you are saving up for your wedding in 5 years. You put $10,000 in an account earning 3% interest compounded monthly. How much money will be in the account at the end of 5 years?

$$A = 10{,}000\left(1 + \frac{0.03}{12}\right)^{12\times5} = 10{,}000\left(1 + \frac{0.03}{12}\right)^{60} = \$11{,}616.17$$

If you want to know the amount of interest paid, A= future value, subtract the principal to get the interest. A-P = Interest

$$Interest = 11{,}616.17 - 10{,}000 = 1{,}616.17$$

Effective rate and continuous compounding.

Effective rate: is used to compare the equivalent annual rate earned in order to compare rates with different compounding periods.

$$R_{eff} = \left(1 + \frac{r}{n}\right)^{n} - 1$$

Compare: Bank A has a savings account that pays 5% compounded quarterly, and Bank B has a savings account that pays 4.75% compounded daily. Which savings account has the higher effective rate?

Use the future value formula, but Treat time like 1 and principal like $1.

Continuous compounding:
There is a limit that you can earn from compounding more and more often: continuous compounding.

$$A = Pe^{rt}$$

A= future amount (future value)
P=principal (present value)
e is a number approximately equal to 2.71828…
r = annual interest rate

t = time in years

Some things will behave like continuous compounding: stock markets, inflation, population growth, etc.

Repeated compounding can never get higher than the value from continuous compounding.

$$A = 10{,}000e^{0.05\times 1} = 10{,}512.71$$

Amortization Tables
Is a table where each compounding period is treated as simple interest and then end value for that period goes to the start of the next period.

Can build in regular (or irregular payments), changing rates or other adjustments.

Payments with compounding interest: formulas.

$$\mathrm{PVA} \quad = \quad \mathrm{PMT}\left[\frac{(1+\mathrm{i})^{n}-1}{\mathrm{i}(1+\mathrm{i})^{n}}\right]$$

where: PVA = present value of the annuity

PMT = payment per period

i = interest rate

n = number of periods

You are accumulating value in the account. Sinking fund. The positive exponent is telling you that you are adding to the account.

The version where your interest and payments are at odds with each other (interest is adding to the balance, but payments are decreasing), looks very similar, but the exponent is negative.

You aren't expected to use these payment formulas, or to memorize them. Either version. We are going to use Excel to avoid them like the plague.

If we want to solve for anything in the formula at all, Excel is there for us.

See Excel for 3 examples.

Credit cards:
Credit card interest compound daily. If you charge something, you immediately get charged interest on a balance (unless you have a specific grace period). And if you make a payment, you get the credit immediately. So if make an amortization table, you need one for each day.

The bills come with these “financial literacy” reminders about accumulating interest and minimum payments.

The relative benefits of savings vs. paying off debts, paying off debts with different interest rates: which is better?