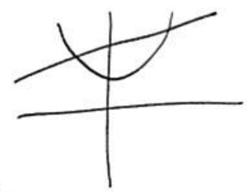
MTH 264, Quiz #3, Spring 2024 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Find the center of mass of the lamina bounded by the graphs $y = x^2 + 1, y = \frac{1}{2}x + 6$ assuming constant density. $x = -2 \quad x = \frac{5}{2}$



$M = \int_{-2}^{5/2} \frac{1}{2}x + 6 - x^2 - 1 \, dx = \int_{-2}^{5/2} -x^2 + \frac{1}{2}x + 5 \, dx = \frac{243}{16}$

$M_x = \frac{1}{2}\int_{-2}^{5/2} (\frac{1}{2}x+6)^2 - (x^2+1)^2 dx = \frac{9963}{160}$

$M_y = \int_{-2}^{5/2} x(-x^2 + \frac{1}{2}x + 5) dx = \int_{-2}^{5/2} -x^3 + \frac{1}{2}x^2 + 5x \, dx = \frac{243}{64}$

$\bar{x} = \frac{M_y}{M} = \frac{\cancel{243}}{64} \cdot \frac{16}{\cancel{243}} = \frac{16}{64} = 1/4 \qquad \bar{y} = \frac{M_x}{M} = \frac{\cancel{9963}^{41}}{\cancel{160}_{10}} \cdot \frac{\cancel{16}}{\cancel{243}} = \frac{41}{10}$

$(1/4, 41/10)$

2. Integrate.

a. $\int \frac{1}{x \ln(x^3)} dx = \int \frac{1}{3x \ln x} dx \qquad u = \ln x \quad du = \frac{1}{x}$

$\frac{1}{3}\int \frac{1}{u} du = \frac{1}{3} \ln u + C$

$= \frac{1}{3} \ln(\ln x) + C$

b. $\int e^{-x} \sec^2(e^{-x}) \, dx \qquad u = e^{-x} \quad du = -e^{-x} dx$

$-\int \sec^2 u \, du = -\tan u + C = -\tan e^{-x} + C$

c. $\int \frac{x^3+1}{x^2+1} dx$

$$\begin{array}{r} x \\ x^2+1 \overline{)\, x^3 \qquad\qquad +1} \\ \underline{-x^3 + \qquad -x} \\ -x+1 \end{array}$$

$\int x - \frac{x}{x^2+1} + \frac{1}{x^2+1} dx = \frac{1}{2}x^2 - \frac{1}{2}\ln|x^2+1| + \tan^{-1} x + C$

d. $\int x \cosh\left(\frac{x^2}{2}\right) dx \qquad u = x^2/2 \quad du = x$

$\sinh(x^2/2) + C$

e. $\int x^2 \sin 3x \, dx$

$-\frac{1}{3}x^2 \cos 3x + \frac{2}{9}x \sin 3x + \frac{2}{27}\cos 3x + C$

±	u	dv
+	x^2	$\sin 3x$
−	$2x$	$-\frac{1}{3}\cos 3x$
+	2	$-\frac{1}{9}\sin 3x$
−	0	$+\frac{1}{27}\cos 3x$