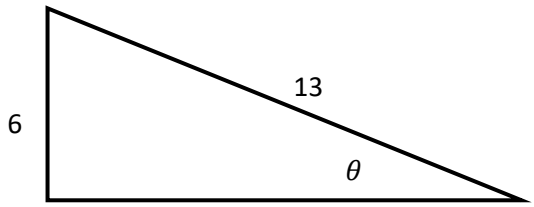
MTH 162, Exam #1, Spring 2025 Name _____

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

Part 1: These questions you will submit answers to in Canvas. Show all work and submit the work with Part 2 of the exam. But you must submit the answers in Canvas to receive credit. Each question/answer will be listed separately. The Canvas question will refer to the number/part to indicate where you should submit which answer. The questions will appear in order (in case there is an inadvertent typo). Correct answers will receive full credit with or without work in this section, but if you don't submit work and clearly label your answers, you won't be able to challenge any scoring decisions for making an error of any kind.

1. Given the dimensions of the triangle below, find the missing side, and all the values of the six trig functions. (14 points)



2. Convert the following angles to radians. (4 points each)
   a. $15°$

   b. $135°$

3. Convert the following angles from radians to degrees. (4 points each)
   a. $\frac{\pi}{8}$

   b. $\frac{13\pi}{10}$

4. At a certain time of day, the angle of elevation of the sun is $40°$. To the nearest foot, find the height of a tree whose shadow is 35 feet long. (12 points)

5. Find the exact value of the six trig functions if the coterminal side of the angle passes through the point $(-1, -3)$. (12 points)

6. Use a reference angle to find the exact value of each of the following. (5 points each)
   a. $\sin\left(-\frac{35\pi}{6}\right)$

   b. $\tan 210°$

7. For each function, state the amplitude, period, phase shift and any vertical shift. (16 points each)
   a. $y = -\frac{1}{2}\cos(2x + \pi)$

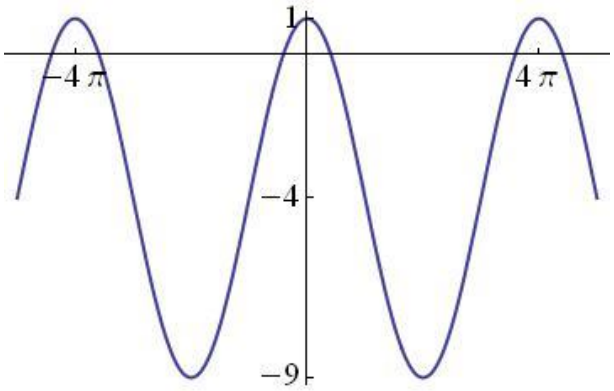
   b. $y = -3\sin 2\pi x + 2$

Part 2: In this section you will record your answers on paper along with your work. After scanning, submit them to a Canvas dropbox as directed. These questions will be graded by hand.

8. Use key points to graph two periods of each function, by hand, using key points. (10 points each)
   a. $y = -2\sin \pi x$

   b. $y = 4\cos\frac{1}{2}x$

9. Write an equation of the graph. (9 points)



10. Graph the functions for 2 periods, by hand, using key points. State the domain of each. (10 points)
    a. $\mathrm{y} = -\tan\left(2x - \frac{\pi}{4}\right)$

11. Find the inverse of the following functions. If the function is not one-to-one, restrict the domain and then find the inverse. State the domain and range of both functions (use the restricted domain where needed). Sketch the function and its inverse on the same graph. (10 points each)
    a. $f(x) = \sqrt[3]{x-4} + 2$

b. $f(x) = x^2 - 4$

12. Apply the following transformations, in order, to the function $f(x) = \log_2(x)$. State the function, the domain and range, and sketch the graph. (10 points)
    a. Reflect horizontally
    b. Shift 3 units to the left
    c. Vertical stretch by 4
    d. Shift up by 5

Some useful formulas:

$$\sin(a+b) = \sin a \cos b + \sin b \cos a$$
$$\sin(a-b) = \sin a \cos b - \sin b \cos a$$
$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$
$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$
$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$
$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

$$\begin{aligned} \sin 2\alpha &= 2\sin\alpha\cos\alpha \\ \cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\ &= 2\cos^2\alpha - 1 \\ &= 1 - 2\sin^2\alpha \\ \tan 2\alpha &= \frac{2\tan\alpha}{1 - \tan^2\alpha} \end{aligned}$$

$$\sin\left(\frac{a}{2}\right) = \pm\sqrt{\frac{(1-\cos a)}{2}}$$
$$\cos\left(\frac{a}{2}\right) = \pm\sqrt{\frac{(1+\cos a)}{2}}$$
$$\tan\left(\frac{a}{2}\right) = \frac{1-\cos a}{\sin a} = \frac{\sin a}{1+\cos a}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$
$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$
$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

$$\cos(a)\cos(b) = \frac{1}{2}\left(\cos(a+b) + \cos(a-b)\right)$$
$$\sin(a)\sin(b) = \frac{1}{2}\left(\cos(a-b) - \cos(a+b)\right)$$
$$\sin(a)\cos(b) = \frac{1}{2}\left(\sin(a+b) + \sin(a-b)\right)$$
$$\cos(a)\sin(b) = \frac{1}{2}\left(\sin(a+b) - \sin(a-b)\right)$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$
$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$