MTH 162, Exam #2, Spring 2025 Name KEY

**Instructions**: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

Part 1: These questions you will submit answers to in Canvas. Show all work and submit the work with Part 2 of the exam. But you must submit the answers in Canvas to receive credit. Each question/answer will be listed separately. The Canvas question will refer to the number/part to indicate where you should submit which answer. The questions will appear in order (in case there is an inadvertent typo). Correct answers will receive full credit with or without work in this section, but if you don't submit work and clearly label your answers, you won't be able to challenge any scoring decisions for making an error of any kind.

1. For $\cos\theta = \frac{12}{13}$, and $\theta$ in Q IV, find each of the following. (6 points each)
   a. $\sin 2\theta$

   $\sin\theta = -\frac{5}{13}$

   $\sin 2\theta = 2\sin\theta\cos\theta =$

   $2 \cdot \left(-\frac{5}{13}\right)\left(\frac{12}{13}\right) = -\frac{120}{169}$

   b. $\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$, $\frac{1-\frac{12}{13}}{-\frac{5}{13}} = \frac{1}{\cancel{13}} \cdot \left(-\frac{\cancel{13}}{5}\right) = -\frac{1}{5}$

2. Solve for all values of the variable in $[0,2\pi)$. (10 points each)
   a. $\sin 2x = \sin x$

   $\sin 2x - \sin x = 0$

   $2\sin x\cos x - \sin x = 0$

   $\sin x(2\cos x - 1) = 0$

   $\sin x = 0 \quad x = 0, \pi$

   $2\cos x - 1 = 0$

   $\cos x = 1/2$

   $x = \pi/3, 5\pi/3$

b. $\sin\left(2x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\sin\alpha = \frac{\sqrt{2}}{2}$

$\alpha = \pi/4,\ 3\pi/4\ \ 9\pi/4,\ 11\pi/4$

$2x - \pi/4 = \pi/4,\ 3\pi/4,\ 9\pi/4,\ 11\pi/4$

$2x = \pi/2,\ \pi,\ 5\pi/2,\ 3\pi$

$x = \pi/4,\ \pi/2,\ 5\pi/4,\ 3\pi/2$

c. $3\cos^2 x = \sin^2 x$

$3\cos^2 x = 1 - \cos^2 x$

$4\cos^2 x = 1$

$\cos^2 x = 1/4$

$\cos x = \pm\frac{1}{2}$

$x = \pi/3,\ 5\pi/3,\ 2\pi/3,\ 4\pi/3$

3. Use identities to find exact values for each of the following. (6 points each)

a. $\cos\frac{5\pi}{12}\cos\frac{\pi}{12} + \sin\frac{5\pi}{12}\sin\frac{\pi}{12}$

$\alpha \quad \beta$

$\alpha - \beta = \frac{5\pi}{12} - \frac{\pi}{12} = \frac{4\pi}{12} = \pi/3$

$\cos(\alpha - \beta) = \cos\pi/3 = 1/2$

b. $\sin 75° = \sin(30° + 45°) = \sin \underset{\pi/6}{30°} \cos \underset{\pi/4}{45°} + \sin \underset{\pi/4}{45°} \cos \underset{\pi/6}{30°}$

$= \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$

c. $\cos 22.5°$ $\cos\left(\frac{\alpha}{2}\right)$ $= \sqrt{\frac{1+\cos 45°}{2}} = \sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}+1}{2}}$

$\alpha = 45°$

d. $\tan\frac{7\pi}{8} = \tan\left(\frac{\alpha}{2}\right) = \frac{1-\cos -\pi/4}{\sin -\pi/4} = \frac{1-1/\sqrt{2}}{-1/\sqrt{2}} = \frac{\sqrt{2}-1}{-1} = 1-\sqrt{2}$

$\alpha = \frac{7\pi}{4} = -\pi/4$

4. Find the exact value of each expression. (8 points each)

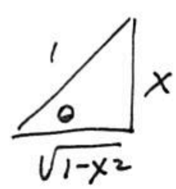
a. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $5\pi/6$

b. $\tan^{-1}(1)$ $= \pi/4$

c. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ $= -\pi/4$

d. $\tan(sin^{-1}(x))$

$\frac{x}{\sqrt{1-x^2}}$



**Part 2**: In this section you will record your answers on paper along with your work. After scanning, submit them to a Canvas dropbox as directed. These questions will be graded by hand.

5. Verify the identities. (10 points each)

a. $(\sec x - \tan x)^2 = \frac{1-\sin x}{1+\sin x}$

$$\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)^2 = \frac{(1-\sin x)^2}{\cos^2 x} = \frac{(1-\sin x)^2}{1-\sin^2 x} = \frac{(1-\sin x)^{\cancel{2}}}{(\cancel{1-\sin x})(1+\sin x)} =$$

$$\frac{1-\sin x}{1+\sin x} \checkmark$$

b. $\sin^4 t - \cos^4 t = 1 - 2\cos^2 t$

$$(\sin^2 t - \cos^2 t)\underbrace{(\sin^2 t + \cos^2 t)}_{=1} = \sin^2 t - \cos^2 t =$$

$$1 - \cos^2 t - \cos^2 t =$$

$$1 - 2\cos^2 t \checkmark$$

c. $\sin\left(x + \frac{3\pi}{2}\right) = -\cos x$

$$\sin x \underbrace{\cos 3\pi/2}_{0} + \underbrace{\sin 3\pi/2}_{-1} \cos x = -\cos x \checkmark$$

d. $\sin^2\frac{\theta}{2} = \frac{\sec\theta - 1}{2\sec\theta}$

$$\sin^2\frac{\theta}{2} = \frac{1}{2}(1-\cos\theta)\cdot\frac{1/\cos\theta}{1/\cos\theta} = \frac{1}{2}\cdot\frac{\left(\frac{1}{\cos\theta}-1\right)}{1/\cos\theta} = \frac{1}{2}\,\frac{\sec\theta-1}{\sec\theta} =$$

$$\frac{\sec\theta-1}{2\sec\theta} \checkmark$$

e. $\sin 2t - \tan t = \tan t\cos 2t$

$$2\sin t\cos t - \frac{\sin t}{\cos t} = \frac{2\sin t\cos^2 t - \sin t}{\cos t} = \frac{\sin t}{\cos t}(2\cos^2 t - 1)$$

$$= \tan t\cos 2t$$

6. Find the domain and range of $y = \sin^{-1}(x-2) + \frac{\pi}{2}$. (8 points)

$\sin^{-1}x \sim [-1, 1]$ $\quad [-\pi/2, \pi/2]$

$+2 \quad +2 \qquad +\pi/2 \quad \pi/2$

Domain: $[1, 3]$ $\qquad [0, \pi]$

of y

Range of y

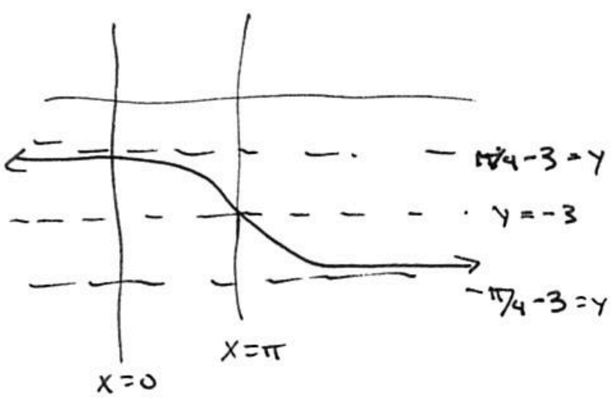
7. Sketch the graph of $f(x) = -\frac{1}{2}\tan^{-1}(x+\pi) - 3$. State the domain and range. (9 points)

$\tan^{-1}x$

D: $(-\infty, \infty)$ $\quad$ R: $(-\pi/2, \pi/2)$

$* -\frac{1}{2}$ $\quad (-\pi/4, \pi/4)$

$-3$ $\quad (-\pi/4 - 3, \pi/4 - 3)$




Domain $(-\infty, \infty)$

Range of f $(-\pi/4 - 3, \pi/4 - 3)$

Some useful formulas:

$$\sin(a+b) = \sin a\cos b + \sin b\cos a$$

$$\sin(a-b) = \sin a\cos b - \sin b\cos a$$

$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$

$$\cos(a-b) = \cos a\cos b + \sin a\sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a\tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a\tan b}$$

$$\begin{aligned}
\sin 2\alpha &= 2\sin\alpha\cos\alpha \\
\cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\
&= 2\cos^2\alpha - 1 \\
&= 1 - 2\sin^2\alpha \\
\tan 2\alpha &= \frac{2\tan\alpha}{1-\tan^2\alpha}
\end{aligned}$$

$$\sin(\frac{a}{2}) = \pm\sqrt{\frac{(1-\cos a)}{2}}$$

$$\cos(\frac{a}{2}) = \pm\sqrt{\frac{(1+\cos a)}{2}}$$

$$\tan(\frac{a}{2}) = \frac{1-\cos a}{\sin a} = \frac{\sin a}{1+\cos a}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

$$\cos(a)\cos(b) = \frac{1}{2}\left(\cos(a+b) + \cos(a-b)\right)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left(\cos(a-b) - \cos(a+b)\right)$$

$$\sin(a)\cos(b) = \frac{1}{2}\left(\sin(a+b) + \sin(a-b)\right)$$

$$\cos(a)\sin(b) = \frac{1}{2}\left(\sin(a+b) - \sin(a-b)\right)$$

$$\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin(A) - \sin(B) = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos(A) - \cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$