MTH 162, Exam #3, Spring 2025 Name ____

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

Part 1: These questions you will submit answers to in Canvas. Show all work and submit the work with Part 2 of the exam. But you must submit the answers in Canvas to receive credit. Each question/answer will be listed separately. The Canvas question will refer to the number/part to indicate where you should submit which answer. The questions will appear in order (in case there is an inadvertent typo). Correct answers will receive full credit with or without work in this section, but if you don't submit work and clearly label your answers, you won't be able to challenge any scoring decisions for making an error of any kind.

1. For each situation below, find all the missing elements of the given triangle. Be sure to check if two triangles exist, and if so, find both. If no triangle exists, explain why. Round lengths and angles (in degrees) to the nearest tenth. (7 points each)
   a. $a = 30, b = 40, A = 20°$

   b. $a = 1.4, b = 2.9, A = 142°$

   c. $a = 3, b = 7, c = 6$

2. When the angle of elevation of the sun is $62°$, a telephone pole that is tilted at an angle of $8°$ away from the sum casts a shadow 20 feet long. Determine the length of the pole to the nearest foot. (8 points)

3. You are on a fishing boat that leaves its pier and heads east. After traveling for 25 miles, there is a report of rough seas directly north, so the captain turns the boat to a bearing of $N40°W$ for 13.5 miles. How far is the boat to the pier, and in which direction would the boat have to sail in order to read port from their current position? (8 points)

4. Find $\left(-\sqrt{3}+i\right)^5$ using DeMoivre's Theorem. (You will not receive partial credit for FOILing.) Write the result in standard form with exact values. (8 points)

5. For $z_1 = 4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ and $z_2 = 12\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$, find the following products or quotients. For standard angles, fully evaluate and give exact answers. You may leave non-standard angles in polar form. (6 points each)
   a. $z_1 z_2$

   b. $\frac{z_2}{z_1}$

6. Use $\vec{u} = \langle 2, -1\rangle, \vec{v} = \langle -3,4\rangle$ to find the following. (5 points each)
   a. $\|\vec{u}\|$

   b. Find $\vec{u} \cdot \vec{v}$

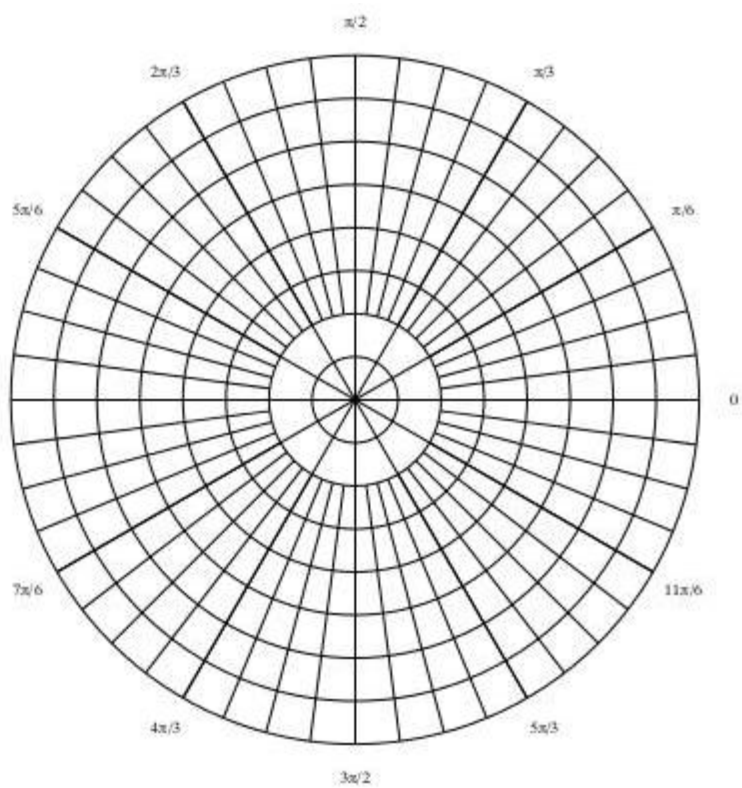
   c. Find the angle between $\vec{u}$ and $\vec{v}$

Part 2: In this section you will record your answers on paper along with your work. After scanning, submit them to a Canvas dropbox as directed. These questions will be graded by hand.

7. Convert the equation $y^2 = 12x$ into polar coordinates. (6 points)

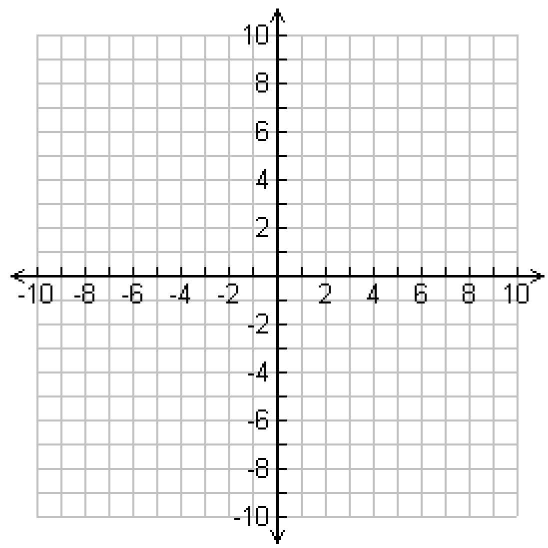
8. Convert the equation $r^2 \sin 2\theta = 6$ into rectangular coordinates. (1 points)

9. Graph the polar equation $r = 1 - 2\cos\theta$ on the polar graph below. Clearly label at least 6 points and show work. (10 points)

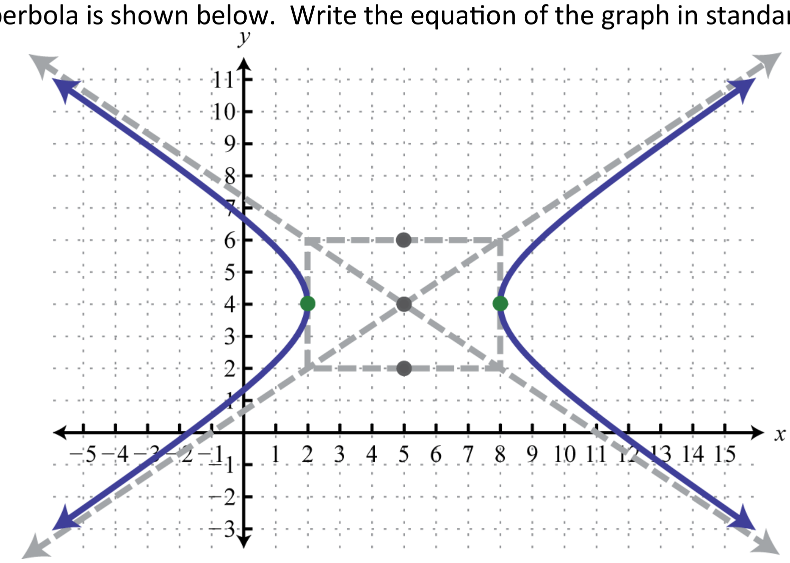


10. Find all the 4th roots of $-2-2\sqrt{3}i$. You may leave your solutions in polar form. (8 points)

11. Graph the equation $(y-2)^2 = 12(x+3)$ on the axes below. Clearly label the focus, vertex and directrix. (8 points)



12. The graph of a hyperbola is shown below. Write the equation of the graph in standard form. (8 points)

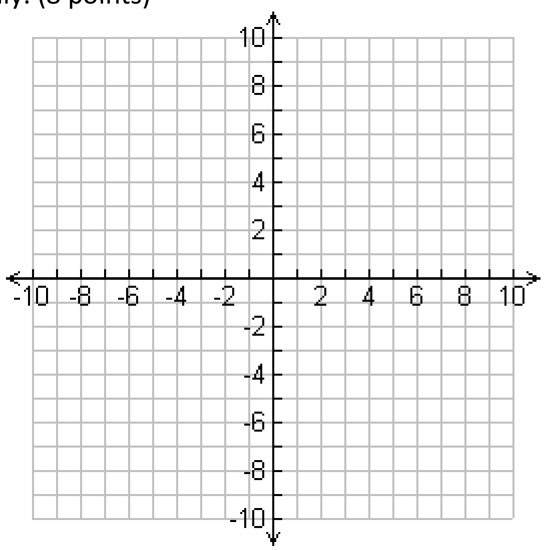


13. An ellipse has the endpoints of the major axis at $(7,9)$ and $(7,3)$, and one focus at $(7,8)$. Find the equation of the ellipse in standard form. (8 points)

14. Identify the type of conic given by the equation $x^2 + 6x - 4y + 1 = 0$ and put the equation in standard form. (8 points)

15. Given the equation $r = \frac{8}{2+4\sin\theta}$, determine the type of conic this represents by finding the eccentricity of the graph. Then use technology to sketch the graph and confirm your results. (8 points)

16. Sketch the graph of the parametric equations $x = 2 + 4\cos t$, $y = -1 + 3\sin t$, by plotting at least 4 points (and labeling them). Then convert the equation back to an equation in x and y only. (8 points)



17. Use $\vec{u} = \langle 3, -2 \rangle$, $\vec{v} = \langle 4,1 \rangle$ to find the following. (5 points each)
    a. Find $\vec{u} + \vec{v}$, then graph $\vec{u}$, $\vec{v}$ and $\vec{u} + \vec{v}$ on the same graph.

    b. Write a unit vector in the direction of v

    c. What is the projection of $\vec{u}$ onto $\vec{v}$?

Some useful formulas:

$$\sin(a+b)=\sin a\cos b+\sin b\cos a$$

$$\sin(a-b)=\sin a\cos b-\sin b\cos a$$

$$\cos(a+b)=\cos a\cos b-\sin a\sin b$$

$$\cos(a-b)=\cos a\cos b+\sin a\sin b$$

$$\tan(a+b)=\frac{\tan a+\tan b}{1-\tan a\tan b}$$

$$\tan(a-b)=\frac{\tan a-\tan b}{1+\tan a\tan b}$$

$$\begin{aligned}\sin 2\alpha &= 2\sin\alpha\cos\alpha\\ \cos 2\alpha &= \cos^2\alpha-\sin^2\alpha\\ &= 2\cos^2\alpha-1\\ &= 1-2\sin^2\alpha\\ \tan 2\alpha &= \frac{2\tan\alpha}{1-\tan^2\alpha}\end{aligned}$$

$$\sin(\frac{a}{2})=\pm\sqrt{\frac{(1-\cos a)}{2}}$$

$$\cos(\frac{a}{2})=\pm\sqrt{\frac{(1+\cos a)}{2}}$$

$$\tan(\frac{a}{2})=\frac{1-\cos a}{\sin a}=\frac{\sin a}{1+\cos a}$$

$$\sin^2\theta=\frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta=\frac{1+\cos 2\theta}{2}$$

$$\tan^2\theta=\frac{1-\cos 2\theta}{1+\cos 2\theta}$$

$$\begin{aligned}\cos(a)\cos(b) &= \frac{1}{2}\big(\cos(a+b)+\cos(a-b)\big)\\ \sin(a)\sin(b) &= \frac{1}{2}\big(\cos(a-b)-\cos(a+b)\big)\\ \sin(a)\cos(b) &= \frac{1}{2}\big(\sin(a+b)+\sin(a-b)\big)\\ \cos(a)\sin(b) &= \frac{1}{2}\big(\sin(a+b)-\sin(a-b)\big)\end{aligned}$$

$$\sin(A)+\sin(B)=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin(A)-\sin(B)=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos(A)+\cos(B)=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos(A)-\cos(B)=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$