MTH 162, Final Exam, Spring 2025   Name ____

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

Part 1: These questions you will submit answers to in Canvas. Show all work and submit the work with Part 2 of the exam. But you must submit the answers in Canvas to receive credit. Each question/answer will be listed separately. The Canvas question will refer to the number/part to indicate where you should submit which answer. The questions will appear in order (in case there is an inadvertent typo). Correct answers will receive full credit with or without work in this section, but if you don't submit work and clearly label your answers, you won't be able to challenge any scoring decisions for making an error of any kind.

1. You are on a fishing boat that leaves its pier and heads east. After traveling for 25 miles, there is a report of rough seas directly north, so the captain turns the boat to a bearing of $N35°W$ for 11 miles. How far is the boat to the pier, and in which direction would the boat have to sail in order to read port from their current position? (10 points)

2. Evaluate the following expressions. (9 points each)
   a. $\frac{20!}{4!16!}$

   b. $\sum_{k=1}^{4}(k-3)(k+2)$

c. $\binom{15}{2}$

3. Find the exact value of the six trig functions if the coterminal side of the angle passes through the point $(2, -5)$. (12 points)

4. For $\cos\theta = \frac{7}{25}$, and θ in Q IV, find each of the following. (7 points each)
   a. $\cos 2\theta$

   b. $\tan\frac{\theta}{2}$

5. Solve the equation $7\cos x = 2\sin^2 x$ for all values of x in $[0,2\pi)$. Use exact values when possible, or round answers to 4 decimal places. (10 points)

6. Use the law of sines and/or the law of cosines to find the missing angles or sides of the following. If there are two triangles, find both. If no triangle is possible, state that. (10 points each)
   a. $a = 4, b = 7, \alpha = 23°$

   b. $a = 5, b = 9, \gamma = 67°$

7. Use $\vec{u} = \langle 4,5 \rangle, \vec{v} = \langle 7, -2 \rangle$ to find the following (7 points each).
   a. $||\vec{u}||$

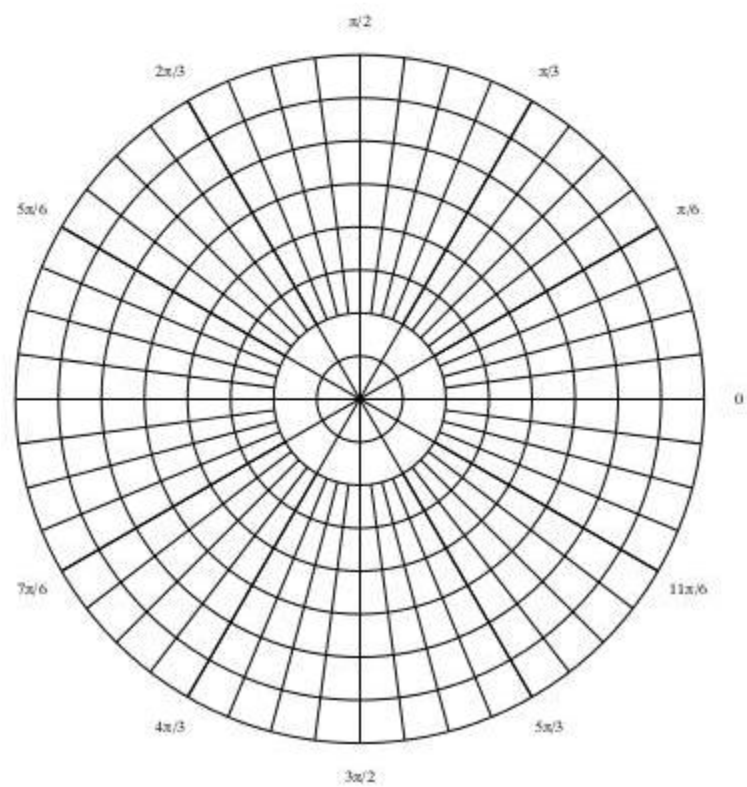
   b. Find $\vec{u} \cdot \vec{v}$

   c. Find the angle between $\vec{u}$ and $\vec{v}$

Part 2: In this section you will record your answers on paper along with your work. After scanning, submit them to a Canvas dropbox as directed. These questions will be graded by hand.

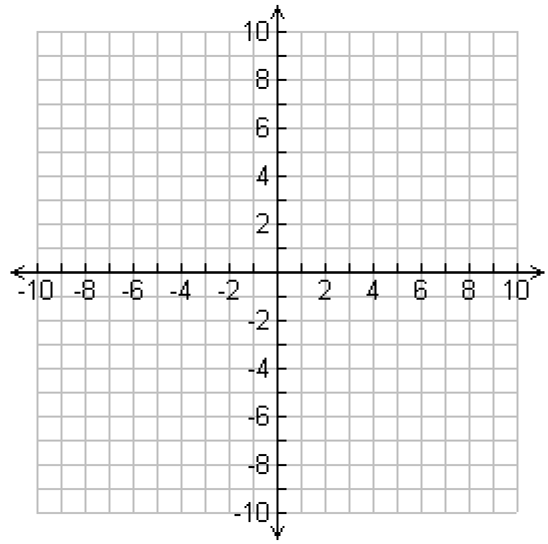
8. Convert the equation $\theta = \frac{2\pi}{3}$ into rectangular coordinates. (8 points)

9. Graph the polar equation $r = 4\cos 3\theta$ on the polar graph below. Clearly label at least 6 points and show work. (12 points)

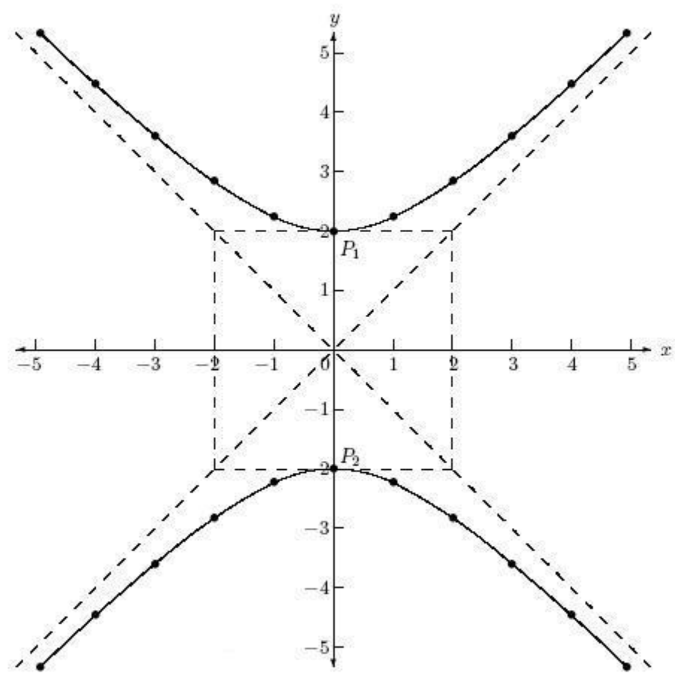


10. Find $(-1+i)^7$ using DeMoivre's Theorem. (You will no receive credit for FOILing.) Write the result in standard form with exact values. (12 points)

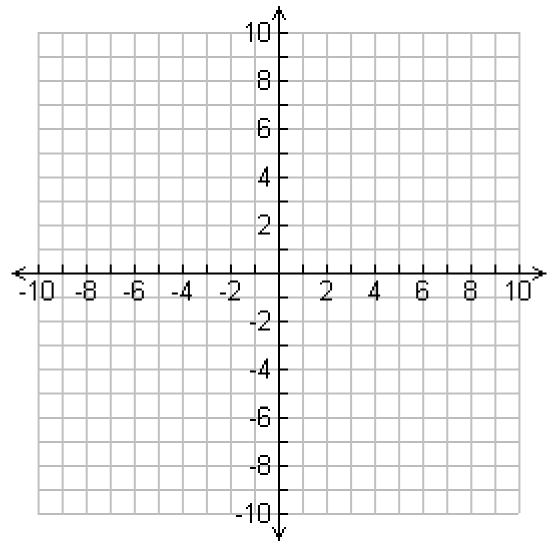
11. Graph the equation $\frac{(x+2)^2}{25} + \frac{(y-3)^3}{9} = 1$ on the axes below. Clearly label the foci, vertices and minor axis endpoints. (10 points)



12. The graph of a hyperbola is shown below. Write the equation of the graph in standard form. (10 points)



13. Sketch the graph of the parametric equations $x = 2t + 3, y = -3t + 1$, by plotting at least 4 points (and labeling them). Use an arrow to indicate the orientation of time. Then convert the equation back to an equation in x and y only.
(10 points)



14. Write the sum of $1 + 8 + 27 + 64 + 125 + \cdots + 729$ in summation notation. (8 points)

15. Use the binomial theorem to expand $(2y - 3)^4$. (12 points)

16. Use key points to graph two periods of the function $y = 3\cos\frac{\pi}{2}x + 1$, by hand. (12 points)

17. Find the exact value of each expression. (7 points each)

a. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

b. $\sec(tan^{-1}(x))$

18. Find the domain and range of $y = -\sin^{-1}(x-1) + \frac{\pi}{4}$. (8 points)

19. Verify the identity. (12 points each)

a. $\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin(\alpha)\cos(\beta)$

b. $\frac{\cos(t+h)-\cos t}{h} = \cos(t)\left(\frac{\cos(h)-1}{h}\right) - \sin(t)\left(\frac{\sin(h)}{h}\right)$

20. Use $\vec{u} = \langle 1, -2 \rangle, \vec{v} = \langle 4,8 \rangle$ to find the following. (7 points each)
    a. Find $\vec{u} + \vec{v}$, then graph $\vec{u}, \vec{v}$ and $\vec{u} + \vec{v}$ on the same graph.

    b. Write a unit vector in the direction of $\vec{u}$

    c. What is the projection of $\vec{u}$ onto $\vec{v}$?

Some useful formulas:

$$\sin(a+b) = \sin a \cos b + \sin b \cos a$$

$$\sin(a-b) = \sin a \cos b - \sin b \cos a$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

$$\begin{aligned} \sin 2\alpha &= 2 \sin\alpha \cos\alpha \\ \cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\ &= 2\cos^2\alpha - 1 \\ &= 1 - 2\sin^2\alpha \\ \tan 2\alpha &= \frac{2\tan\alpha}{1-\tan^2\alpha} \end{aligned}$$

$$\sin(\frac{a}{2}) = \pm\sqrt{\frac{(1-\cos a)}{2}}$$

$$\cos(\frac{a}{2}) = \pm\sqrt{\frac{(1+\cos a)}{2}}$$

$$\tan(\frac{a}{2}) = \frac{1-\cos a}{\sin a} = \frac{\sin a}{1+\cos a}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

$$\cos(a)\cos(b) = \frac{1}{2}\big(\cos(a+b) + \cos(a-b)\big)$$

$$\sin(a)\sin(b) = \frac{1}{2}\big(\cos(a-b) - \cos(a+b)\big)$$

$$\sin(a)\cos(b) = \frac{1}{2}\big(\sin(a+b) + \sin(a-b)\big)$$

$$\cos(a)\sin(b) = \frac{1}{2}\big(\sin(a+b) - \sin(a-b)\big)$$

$$\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin(A) - \sin(B) = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos(A) - \cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$