MTH 162, Homework #3, Spring 2025 Name ____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Verify each identity. Work only one side at a time.
   a. $\tan x \csc x \cos x = 1$
   b. $\frac{\cos\theta \sec\theta}{\cot\theta} = \tan\theta$
   c. $\frac{\sin t}{\tan t} + \frac{\cos t}{\cot t} = \sin t + \cos t$
   d. $\frac{\sec x - \csc x}{\sec x + \csc x} = \frac{\tan x - 1}{\tan x + 1}$
   e. $\frac{1+\cos t}{1-\cos t} = (\csc t + \cot t)^2$
   f. $(\tan^2\theta + 1)(\cos^2\theta + 1) = \tan^2\theta + 2$
   g. $\frac{\cos(x+h) - \cos x}{h} = \cos x\left(\frac{\cos h - 1}{h}\right) - \sin x\left(\frac{\sin h}{h}\right)$
   h. $(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$
   i. $\sin 4t = 4\sin t\cos^3 t - 4\sin^3 t\cos t$
   j. $\tan\frac{x}{2} + \cot\frac{x}{2} = 2\csc x$
   k. $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$
   l. $\sin t\tan t = \frac{1-\cos^2 t}{\cos t}$
   m. $1 - \frac{\cos^2 x}{1+\sin x} = \sin x$
   n. $\frac{\tan x + \tan y}{1 - \tan x\tan y} = \frac{\sin x\cos y + \cos x\sin y}{\cos x\cos y - \sin x\sin y}$
   o. $\frac{\cos^2 t + 4\cos t + 4}{\cos t + 2} = \frac{2\sec t + 1}{\sec t}$
   p. $\frac{\sin x - \cos x + 1}{\sin x + \cos x - 1} = \frac{\sin x + 1}{\cos x}$
   q. $\frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta} = \tan\alpha - \tan\beta$
   r. $1 - \tan^2 x = \frac{\cos 2x}{\cos^2 x}$
   s. $2\tan\frac{\alpha}{2} = \frac{\sin^2\alpha + 1 - \cos^2\alpha}{\sin\alpha(1+\cos\alpha)}$

2. Use appropriate identities (sum and difference, double angle, half angle and power-reducing) to fin exact values for each expression.
   a. $\cos\left(\frac{3\pi}{4} - \frac{\pi}{6}\right)$
   b. $\cos 75°$
   c. $\frac{\tan\frac{\pi}{5} - \tan\frac{\pi}{30}}{1 + \tan\frac{\pi}{5}\tan\frac{\pi}{30}}$
   d. $\frac{2\tan\frac{\pi}{8}}{1 - \tan^2\frac{\pi}{8}}$
   e. $\tan\frac{3\pi}{8}$
   f. $\cos\left(\frac{5\pi}{18}\right)\cos\left(\frac{\pi}{9}\right) + \sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{\pi}{9}\right)$
   g. $\tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$
   h. $2\sin 15°\cos 15°$
   i. $\sin 105°$

3. Find the exact value of each expression by hand.
   a. $\sin\left(\cos^{-1}\frac{1}{2}+\sin^{-1}\frac{3}{5}\right)$
   b. $\sin\left(2\,sin^{-1}\frac{\sqrt{3}}{2}\right)$
   c. $\cos\left(tan^{-1}\frac{4}{3}+cos^{-1}\frac{5}{13}\right)$
   d. $\cos^2\left(\frac{1}{2}\sin^{-1}\frac{3}{5}\right)$

4. Write each trigonometric expression without trig functions. Sketch a graph.
   a. $\cos(sin^{-1}x - cos^{-1}y)$
   b. $\sin(tan^{-1}x - sin^{-1}y)$

5. Solve the equations for all angles in $[0,2\pi)$. Give exact answers in terms of π.
   a. $\sin x = \frac{\sqrt{3}}{2}$
   b. $\tan\frac{x}{2} = \sqrt{3}$
   c. $2\sin^2 x - \sin x - 1 = 0$
   d. $\sec^2 x - 2 = 0$
   e. $\cos 2x = \cos x$
   f. $5\sin x = 2\cos^2 x - 4$
   g. $2\cos x + \sqrt{3} = 0$
   h. $\sin\left(2x+\frac{\pi}{6}\right) = \frac{1}{2}$
   i. $\cos^2 x + 2\cos x - 3 = 0$
   j. $\sin x + 2\sin x\cos x = 0$
   k. $\sin x + \cos x = 1$
   l. $2\cos^3 x + \cos^2 x - 2\cos x - 1 = 0$

6. Solve the equations for all angles in $[0,2\pi)$. Round answers to four decimal places.
   a. $\cos x = -\frac{2}{5}$
   b. $4\tan^2 x - 8\tan x + 3 = 0$
   c. $5\sin^2 x - 1 = 0$
   d. $2\sin 3x + \sqrt{3} = 0$