MTH 162, Homework #7, Spring 2025 Name _____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Graph the parametric plane curve on the indicated interval. Use arrows to show the orientation of the curve. Write the equation in non-parametric form.
   a. $x = t + 2, y = t^2, -2 \le t \le 2$
   b. $x = \sqrt{t}, y = t - 1, t \ge 0$
   c. $x = 2t, y = |t - 1|, (-\infty, \infty)$
   d. $x = 2\sin t\,, y = 2\cos t\,, 0 \le t \le 2\pi$
   e. $x = 2\cos t\,, y = 3\sin t\,, 0 \le t \le 2\pi$
   f. $x = \sec t\,, y = \tan t\,, -\frac{\pi}{2} < t < \frac{\pi}{2}$

2. Write the parametric equations for the given equation or fitting the given description.
   a. $y = 4x - 3$
   b. $y = x^2 - 3$
   c. Circle with center at $(3,5)$, Radius is 6
   d. Ellipse with center at $(-2,3)$, major axis is 10, minor axis is 4
   e. Hyperbola with center at origin, and vertices at $(0, \pm 4)$, foci at $(0, \pm 5)$
   f. Line passing through $(-2,4)$ and $(1,7)$

3. Write the first five terms of the sequence, starting with $n = 0$ if no initial term is given.
   a. $a_n = 4n - 1$
   b. $a_n = (-1)^{n+1}(n + 4)$
   c. $a_n = \frac{(-1)^n}{2^n + 1}$
   d. $a_n = 2a_{n-1} + 3, a_1 = 4$
   e. $a_n = \frac{(n+1)!}{n^2}$
   f. $a_n = (-3)^n$
   g. $a_n = \frac{3n}{n+5}$
   h. $a_n = a_{n-1} + 5, a_1 = 7$
   i. $a_n = \frac{n^2}{n!}$

4. Find the indicated sum.
   a. $\sum_{i=1}^{6} 5i$
   b. $\sum_{i=2}^{4} \left(-\frac{1}{3}\right)^i$
   c. $\sum_{i=1}^{5} i^3$
   d. $\sum_{i=0}^{4} \frac{(-1)^{i+1}}{(i+1)!}$

5. Write each sum in summation notation.
   a. $1 + 4 + 9 + \cdots + 225$
   b. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots + \frac{14}{15}$
   c. $5 + 7 + 9 + 11 + \cdots + 31$
   d. $2 + 4 + 8 + 16 + \cdots + 2048$
   e. $\frac{1}{9} + \frac{2}{81} + \frac{3}{9^3} + \cdots + \frac{n}{9^n}$

6. Write the first five terms of a geometric sequence, starting at $n = 1$, given the following. Write the explicit formula for each sequence.
   a. $a_1 = 5, r = 3$
   b. $a_1 = 24, r = \frac{1}{3}$
   c. $a_n = -5a_{n-1}, a_1 = -6$
   d. $a_1 = 1000, r = -\frac{1}{2}$

7. Write a formula for the sequence.
   a. $3, 12, 48, 192, \ldots$
   b. $5, -1, \frac{1}{5}, -\frac{1}{25}, \ldots$
   c. $12, 6, 3, \frac{3}{2}, \ldots$

8. Use mathematical induction to prove the following.
   a. $\sum_{i=1}^{n}(2i-1) = n^2$
   b. $\sum_{i=1}^{n}\frac{1}{i(i+1)} = \frac{n}{n+1}$
   c. $\sum_{i=1}^{n} 2^i = 2^{n+1} - 2$
   d. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

9. Use the binomial theorem to expand the following powers of the given binomial.
   a. $(x+2)^3$
   b. $(2x^3+1)^4$
   c. $(5x-1)^4$
   d. $(3x-y)^5$

10. Find the indicated term of the polynomial expansion
   a. $(2x+y)^6$, third term
   b. $\left(x-\frac{1}{2}\right)^{22}$, 15th term
   c. $(x^2+y^2)^9$, fourth term
   d. $(x^3+x^{-2})^4$, third term