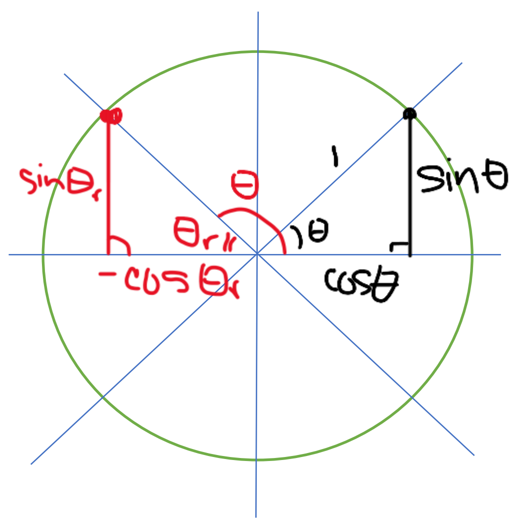
2/13/2025

(Tuesday's class was cancelled due to snow)
Picking up where we left off last week…
Defining the six trig functions for the unit circle
Trig Functions of any Angle/Reference angles, graphs
Trig Identities?

Note: see unit circle handout in Canvas for a reference.

We had left off at extending the first quadrant trig functions to 90-degrees and 0-degrees.

What happens if we have angles that are obtuse (in the second quadrant of the unit circle).



The point of intersection on the unit circle, $(x, y) = (\cos\theta, \sin\theta)$

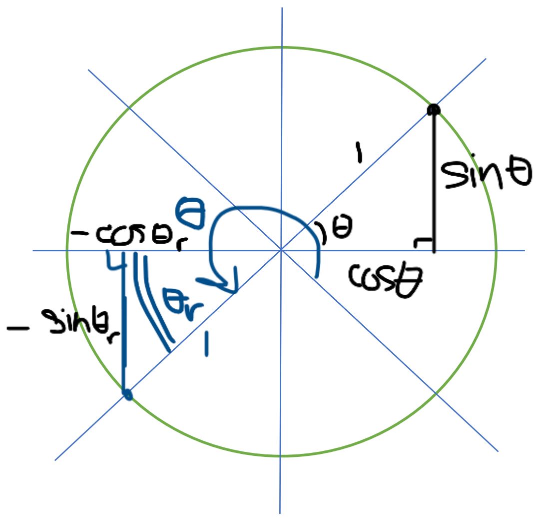
In the second quadrant, θ_r is the reference angle, and the value is $\pi - \theta$ $(180° - \theta)$
The resulting reference angle is an acute angle, so we can apply the values we got for that angle from Quadrant I.

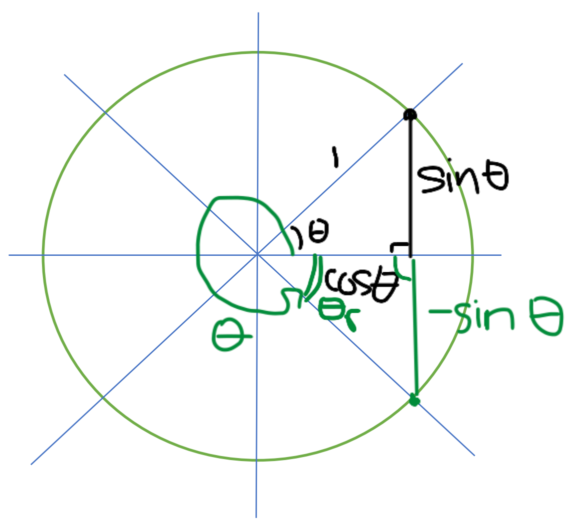
For example, suppose we want to find the sine and cosine values of $135° = \frac{3\pi}{4}$
The reference angle would be $180° - 135° = 45°\ or\ \pi - \frac{3\pi}{4} = \frac{\pi}{4}$

$$\sin(45°) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\cos(45°) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\sin(135°) = \sin\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}}$$
$$\cos(135°) = \cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$





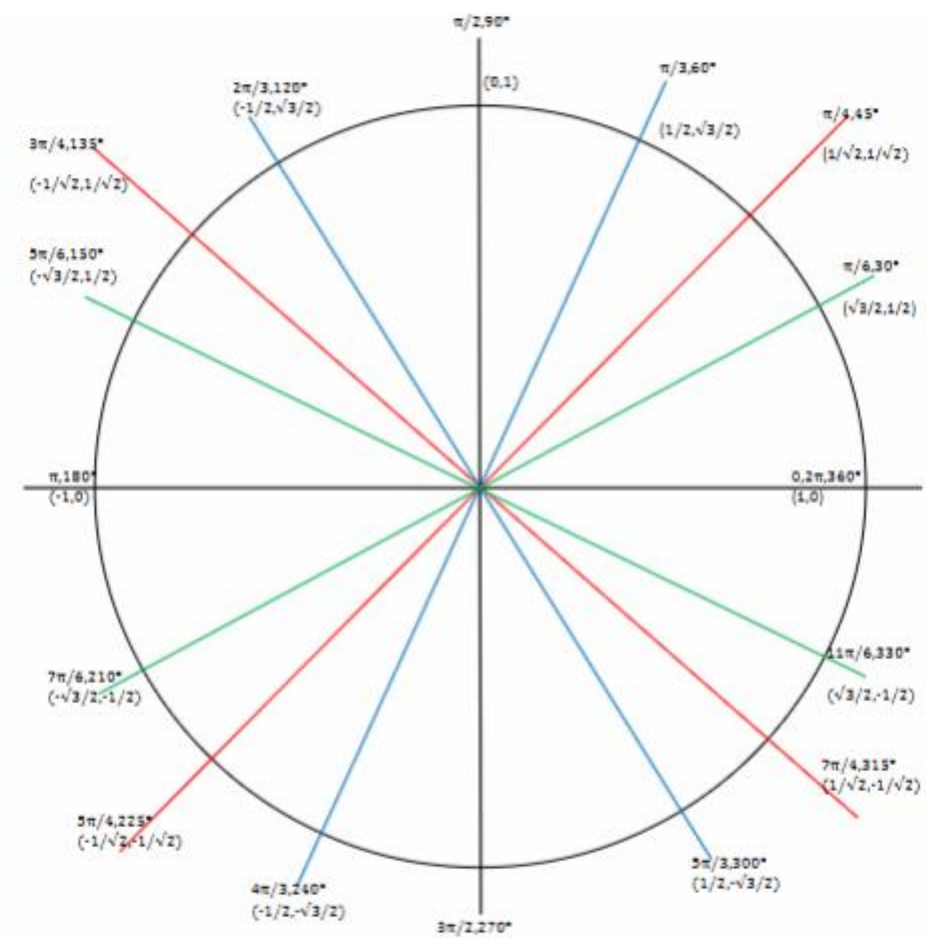
In QIII, the angle θ is between 180° and 270° (or π and $\frac{3\pi}{2}$). The reference angle is $\theta - \pi, \theta - 180°$. Both sine and cosine are negative.

In QIV, the angle θ is between $270°$ and $360°$ (or negative) $(or\,\frac{3\pi}{2}$ and $2\pi)$. The reference angle is $360° - \theta$ or $2\pi - \theta$. The cosine is positive and the sine is negative.

Angle in	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
Q1	$+$	$+$	$+$
Q2	$+$	$-$	$-$
Q3	$-$	$-$	$+$
Q4	$-$	$+$	$-$

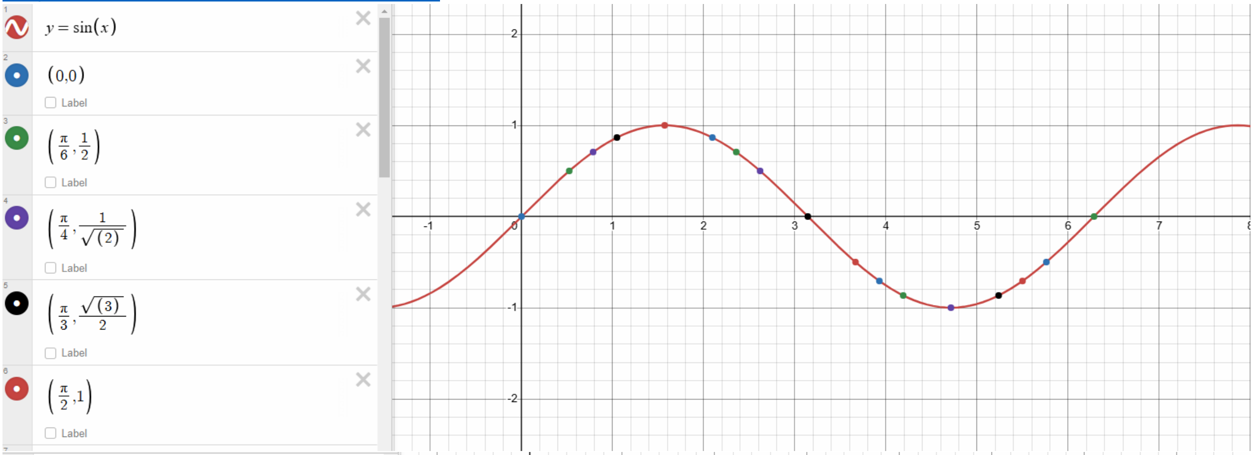
$$\frac{\sin(\theta)}{\cos(\theta)} = \frac{\left(\frac{opp}{hyp}\right)}{\frac{adj}{hyp}} = \frac{opp}{hyp} \times \frac{hyp}{adj} = \frac{opp}{adj} = \tan(\theta)$$

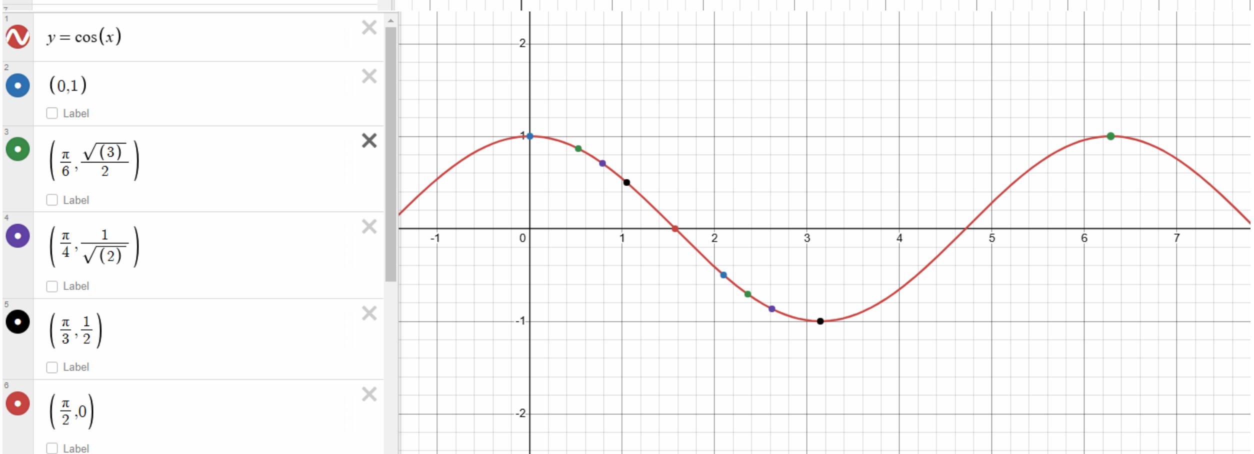
ALL STUDENTS TAKE CALCULUS (which quadrant has positive values for the three trig functions)
ALL = all the trig functions are positive
STUDENTS = sine function is positive and cosine and tangent are negative
TAKE = tangent function that is positive, and cosine and sine are negative
CALCULUS = cosine function is positive, and sine and tangent are negative

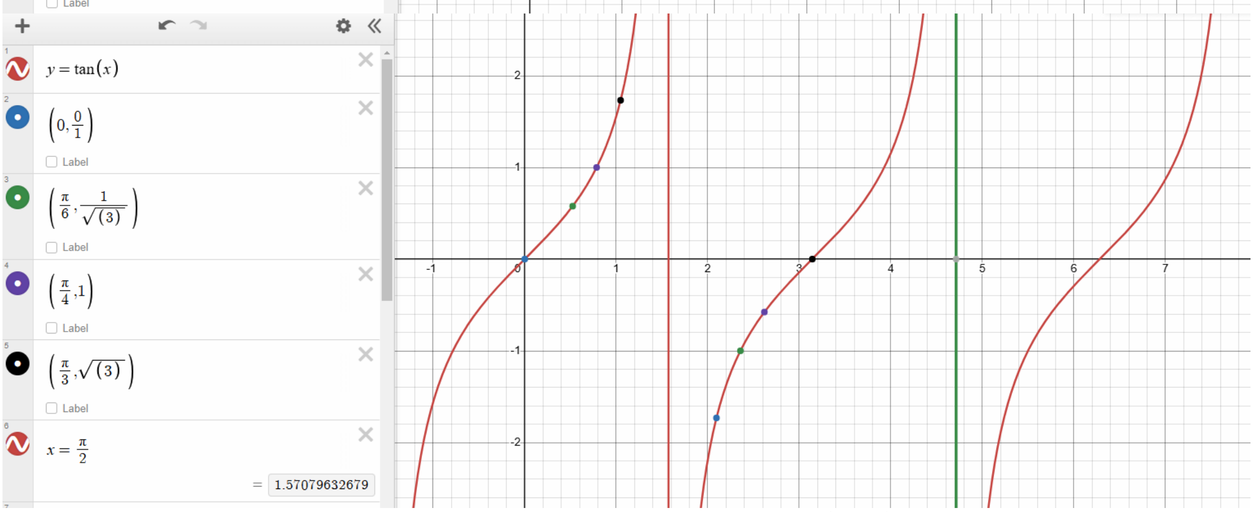


Use this as a reference to plot the values of $y = \sin(x)$, $y = \cos(x)$, $y = \tan(x)$.

https://www.desmos.com/calculator

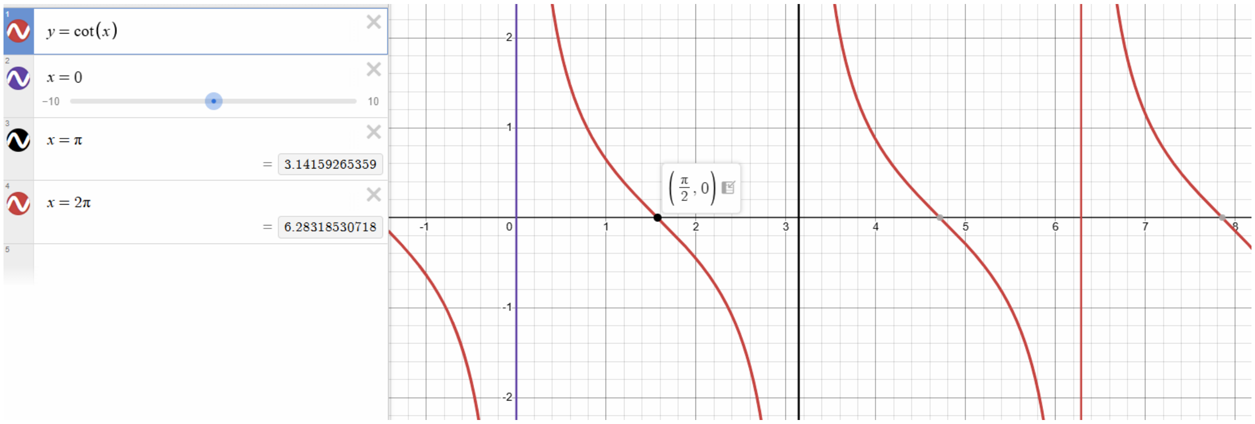




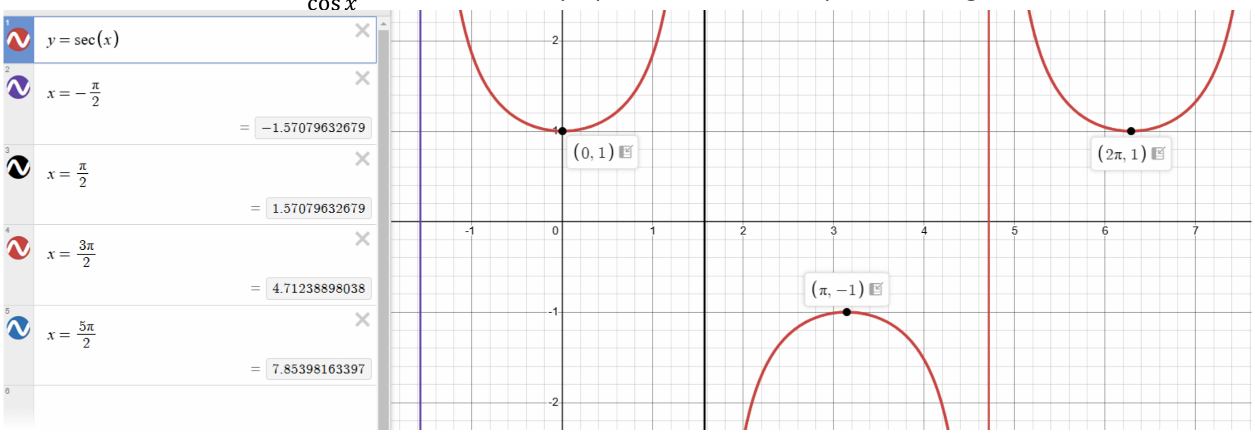


Asymptotes for tangent are where cosine is 0, so odd multiples of $\frac{\pi}{2}$ $\left(\frac{(2n+1)\pi}{2}\right)$ are vertical asymptotes

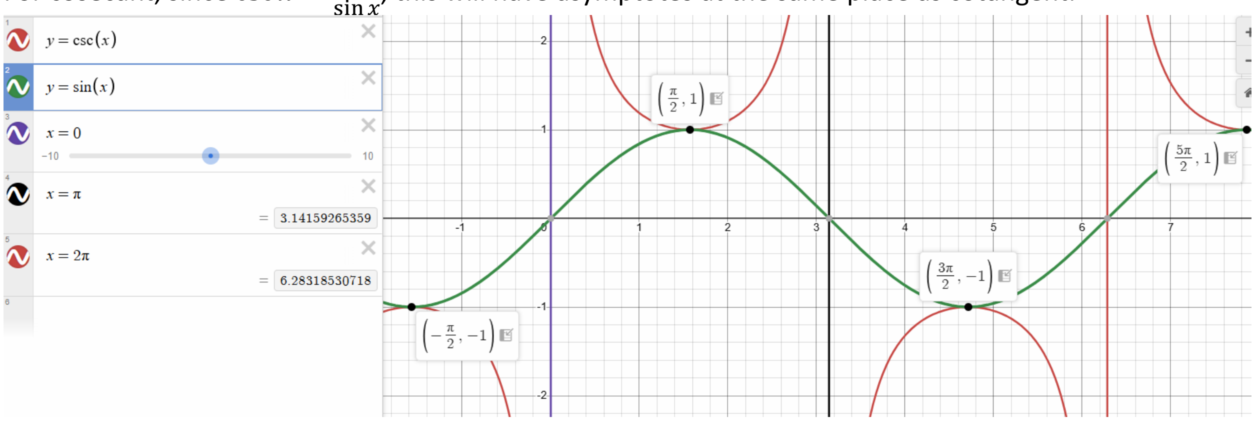
For cotangent, since $\tan x = \frac{\sin x}{\cos x}$, then $\cot x = \frac{\cos x}{\sin x}$. That means it is 0 when cosine is 0, and has a vertical asymptote when sine is 0 (that will be at whole multiple of π).



For secant, since $\sec x = \frac{1}{\cos x}$, this will have asymptotes at the same place as tangent.



For cosecant, since $\csc x = \frac{1}{\sin x}$, this will have asymptotes at the same place as cotangent.



What is the domain and range for each function:

Function	Domain	Range
$\sin(x)$	All real numbers $(-\infty, \infty)$	$[-1,1]$
$\cos(x)$	All real numbers $(-\infty, \infty)$	$[-1,1]$
$\tan(x)$	$\left\{x \middle\| x \neq \frac{(2n+1)\pi}{2}\right\}$	All real numbers $(-\infty, \infty)$
$\cot(x)$	$\{x \| x \neq k\pi\}$	All real numbers $(-\infty, \infty)$
$\sec(x)$	$\left\{x \middle\| x \neq \frac{(2n+1)\pi}{2}\right\}$	$(-\infty, -1] \cup [1, \infty)$

$\csc(x)$	$\{x \mid x \neq k\pi\}$	$(-\infty, -1] \cup [1, \infty)$

Trigonometric Identities

$$\sin(x) = \frac{1}{\csc(x)}, \csc(x) = \frac{1}{\sin(x)}$$

$$\cos(x) = \frac{1}{\sec(x)}, \sec(x) = \frac{1}{\cos(x)}$$

$$\tan(x) = \frac{1}{\cot(x)}, \cot(x) = \frac{1}{\tan(x)}$$

$$\tan(x) = \frac{\sin(x)}{\cos(x)}, \cot(x) = \frac{\cos(x)}{\sin(x)}$$

Complementary angle identities

$$\sin(x) = \cos\left(\frac{\pi}{2} - x\right) = \cos(90° - x)$$

$$\cos(x) = \sin\left(\frac{\pi}{2} - x\right)$$

$$\tan(x) = \cot\left(\frac{\pi}{2} - x\right)$$

$$\cot(x) = \tan\left(\frac{\pi}{2} - x\right)$$

$$\sec(x) = \csc\left(\frac{\pi}{2} - x\right)$$

$$\csc(x) = \sec\left(\frac{\pi}{2} - x\right)$$

Pythagorean identities

$$x^2 + y^2 = r^2$$

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \frac{y^2}{x^2} = \frac{r^2}{x^2}$$

X=adj, y=opp, r=hyp

$$1 + \tan^2\theta = \sec^2\theta$$

$$\frac{x^2}{y^2} + 1 = \frac{r^2}{y^2}$$

$$\cot^2\theta + 1 = \csc^2\theta$$

Properties of symmetry in the trig functions.

$\sin(x)$, $\tan(x)$, $\cot(x)$, $\csc(x)$ are odd functions, $\sin(-x) = -\sin(x)$

$\cos(x)$, $\sec(x)$ are even functions, $\cos(-x) = \cos(x)$

Any trig function repeats after $2\pi\ radians$

$$\sin(x + 2\pi) = \sin(x)$$
$$\cos(x + 2\pi) = \cos(x)$$

Tangent and cotangent repeat after π radians

$$\tan(x + \pi) = \tan(x)$$
$$\cot(x + \pi) = \cot(x)$$

Moving the quiz 2 back to Friday.