2/18/2025

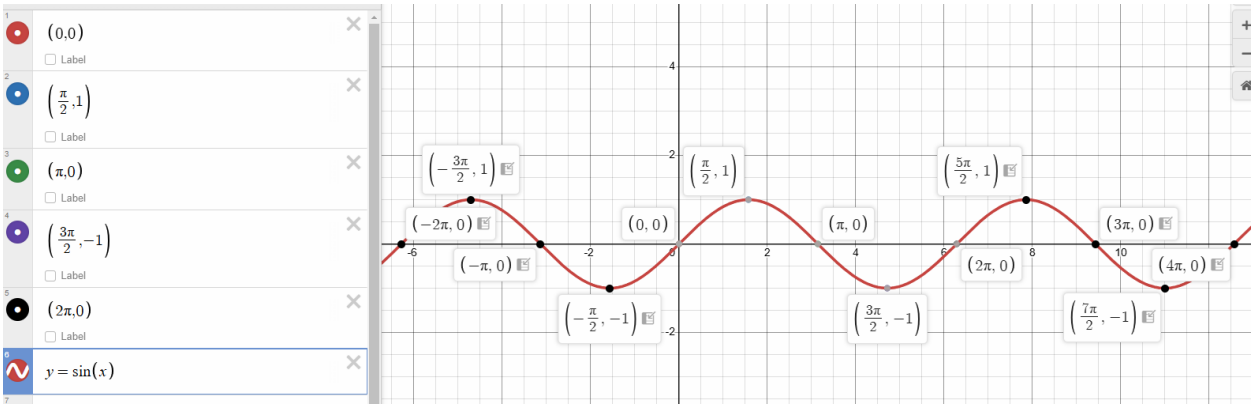
Graphs of Trig Functions
Properties and Transformations

Sine Function

Plot keypoints: Intercepts (x) and minimum and maximum (start with a single cycle) and then repeat that pattern.

Typically for a standard sine function we plot angle values: $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$.

$$(0,0), \left(\frac{\pi}{2}, 1\right), (\pi, 0), \left(\frac{3\pi}{2}, -1\right), (2\pi, 0)$$



Sinusoidal function (they are like sine)

Amplitude of the sine function (sin(x)) is 1. 1 is the distance between the midpoint of the wave and the peak (or valley) of the wave. The maximum is 1, the minimum is -1 and the midpoint is 0.

One way of finding the amplitude is to find the distance between the maximum and the minimum and then divide by 2.

Period: the period of a standard sine function is 2π. Horizontal stretches or compressions we will alter the period. $T = \frac{2\pi}{B} = \frac{2\pi}{\omega}$

Phase shift: this is result of a horizontal shift. ϕ, φ

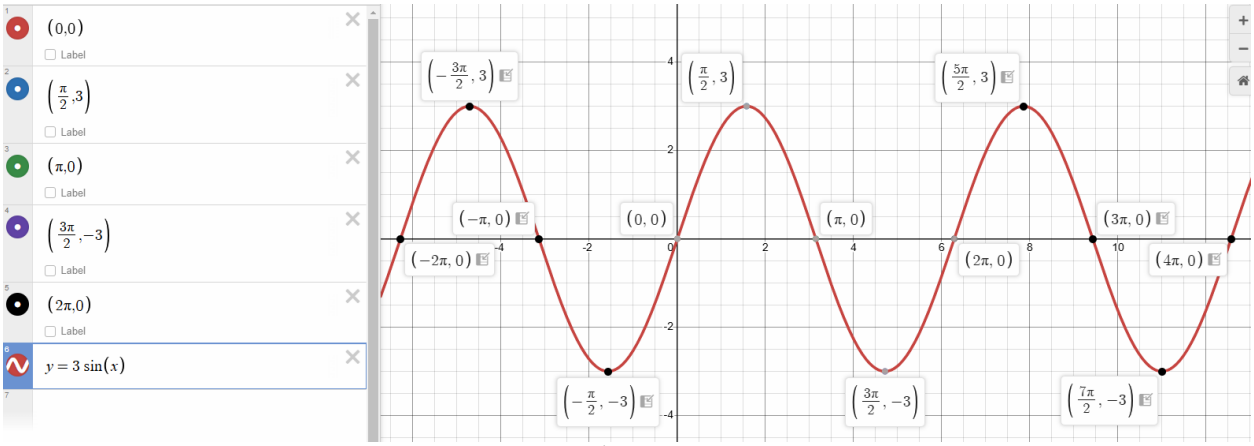
In general: $g(x) = A\sin(Bx + C) + D$

A is changing the amplitude (this is the vertical stretch/compression)
B is the horizontal stretch or compression
$-\frac{C}{B}$ is the phase shift
D is the vertical shift (changes the midpoint)

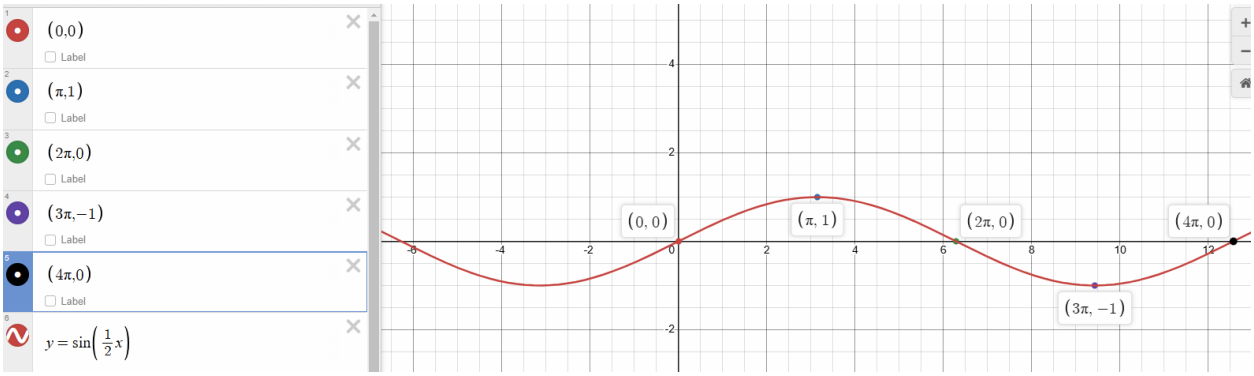
Examples.

We looked at $f(x) = \sin(x)$ see above.

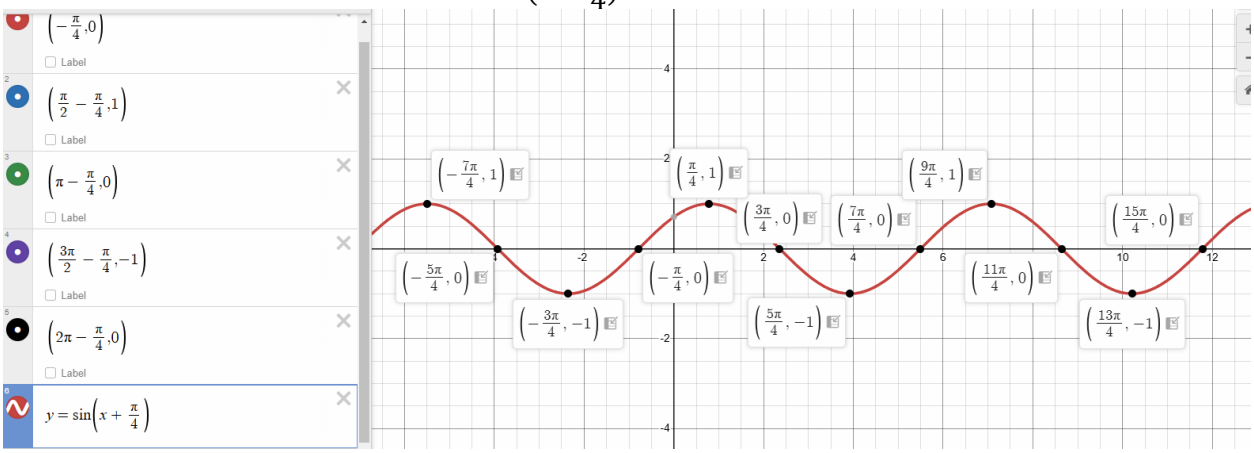
$$g(x) = 3\sin(x) \quad eg. A\sin x$$



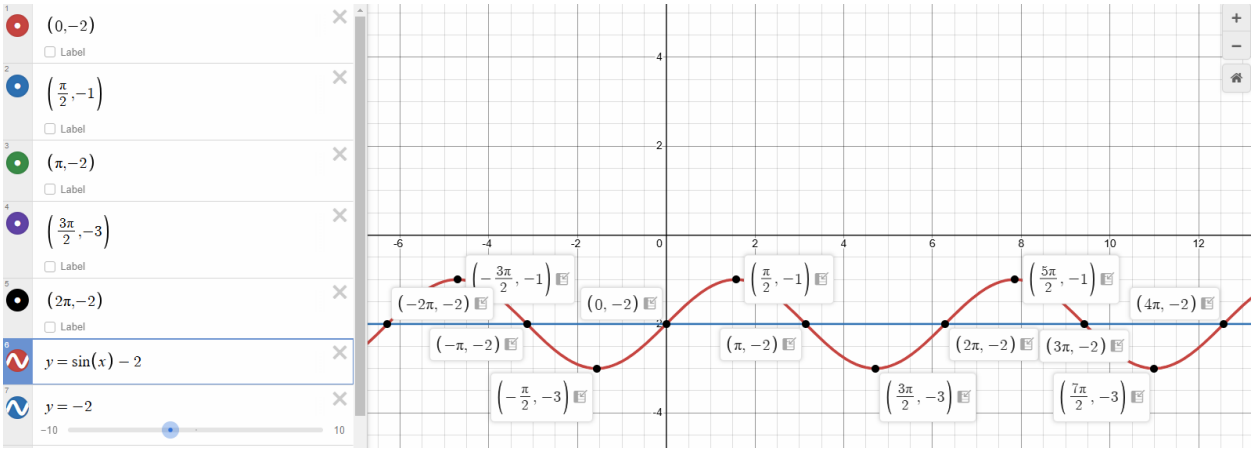
$$h(x) = \sin\left(\frac{1}{2}x\right) \quad eg. \ \sin Bx$$



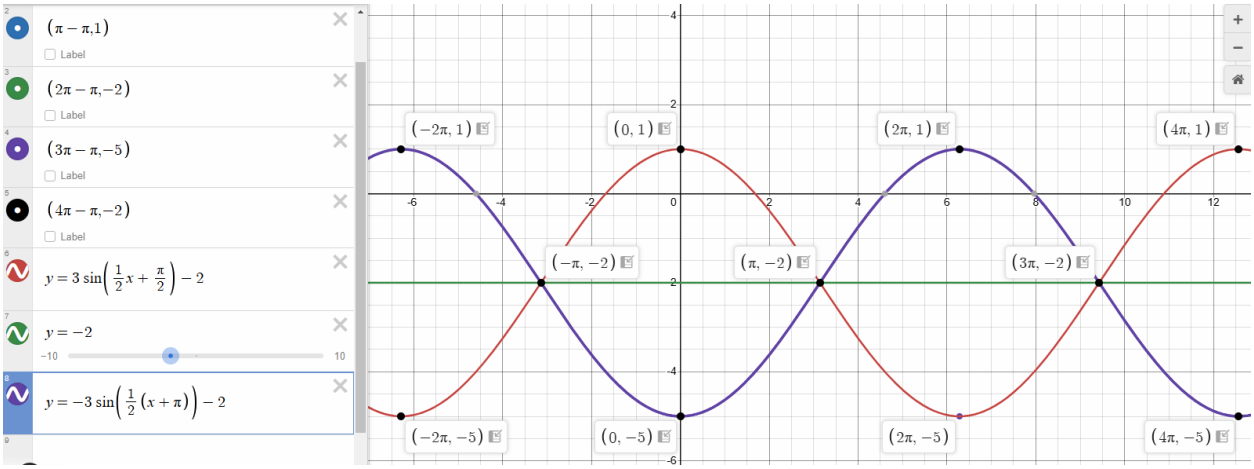
$$k(x) = \sin\left(x + \frac{\pi}{4}\right), \qquad eg. \sin(x + \phi)$$



$$p(x) = \sin(x) - 2, eg. \sin(x) + D$$



$$q(x)=3\sin\left(\frac{1}{2}x+\frac{\pi}{2}\right)-2$$

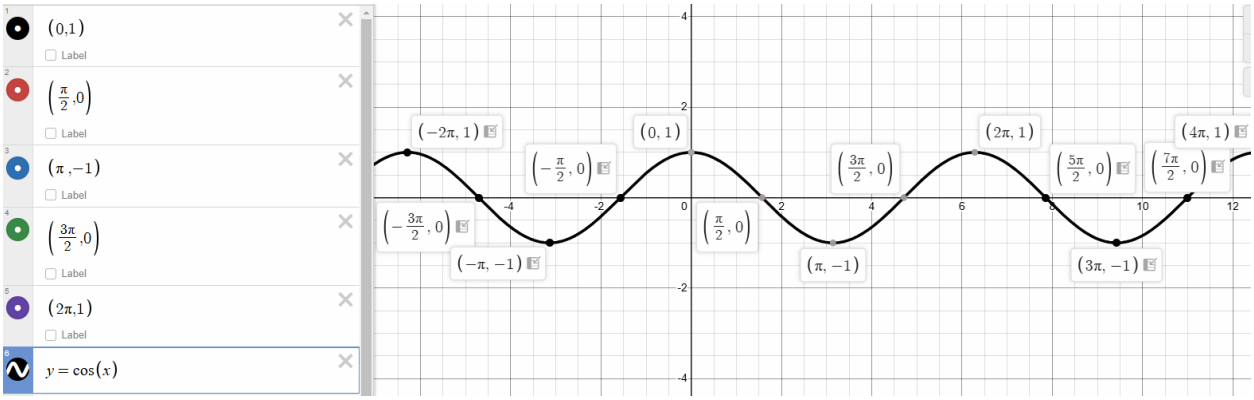


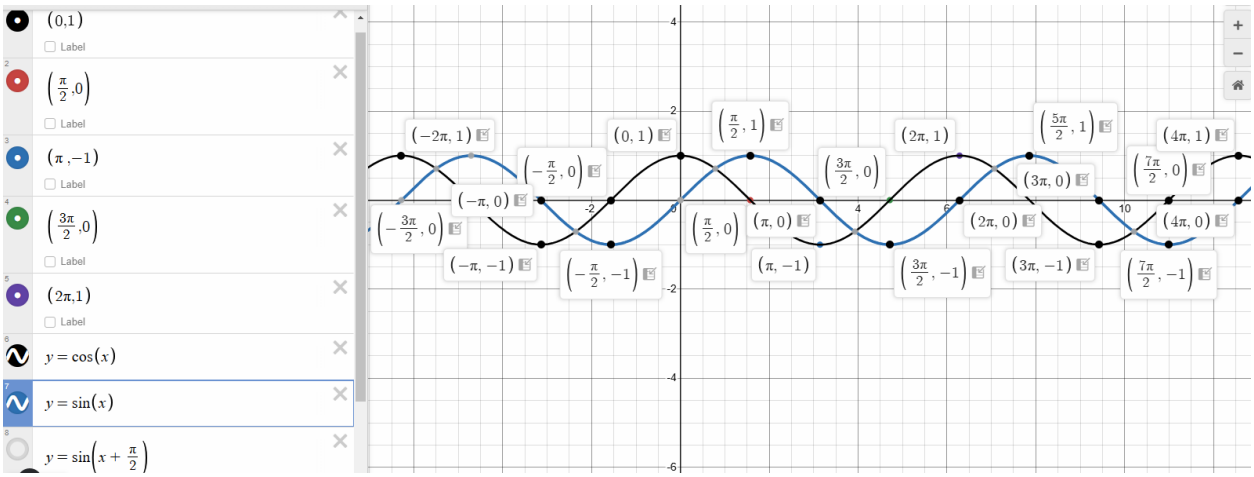
When you plot these by hand, identify the transformation, apply the transformation to the keypoints, and then plot the keypoints, connect the dots to get a smooth curve, extend for the required number of cycles.

Cosine function will work basically the same way.

$$(0,1),\left(\frac{\pi}{2},0\right),(\pi,-1),\left(\frac{3\pi}{2},0\right),(2\pi,1)$$

The base amplitude is still 1, the base period is still 2π.
Cosine is even, so horizontal reflections do nothing (they don't become vertical reflections as they do with sine function).

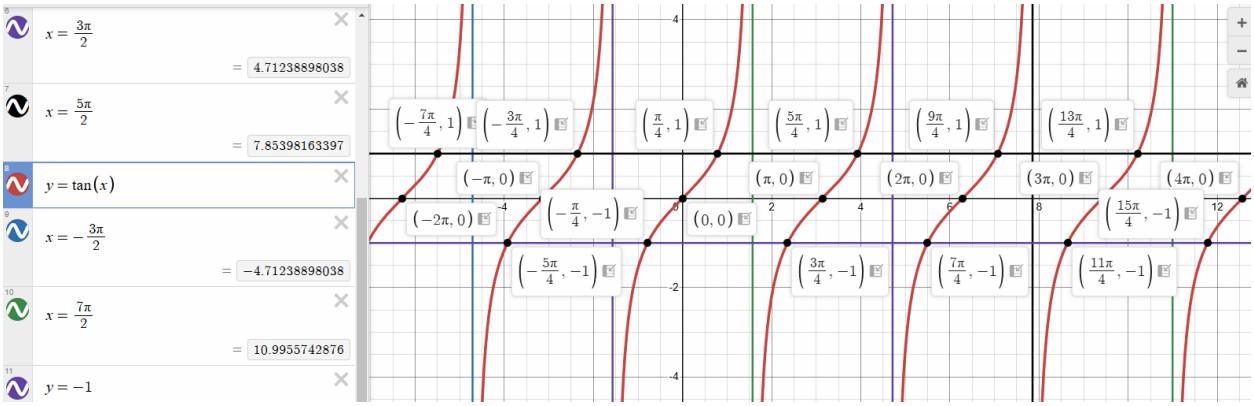


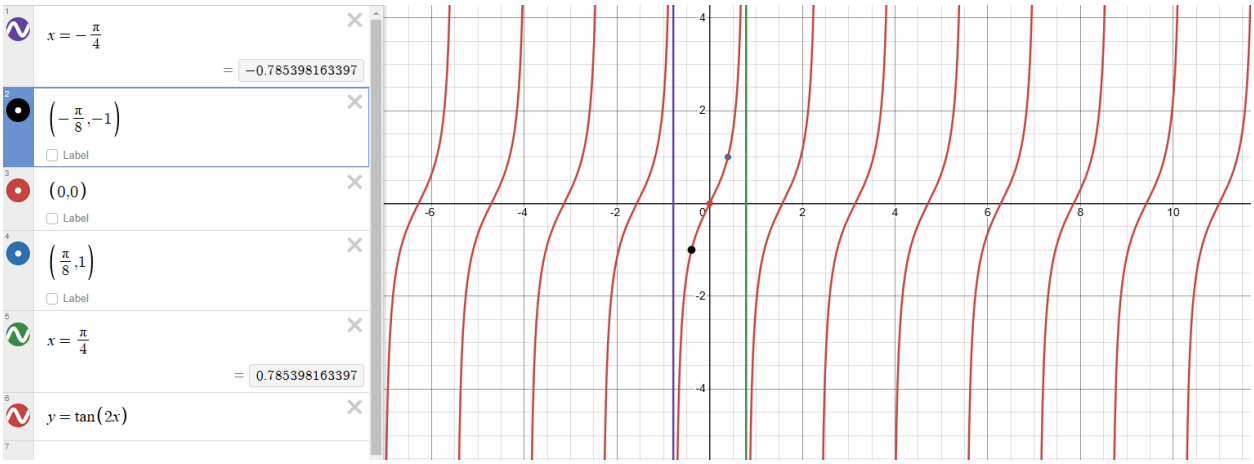


Sine and cosine are the same curve but with a phase shift.

How to plot the tangent function.
The period of tangent is π not 2π. Because the cycle will repeat more quickly, we need to use more closely spaced keypoints. Typically, we use multiples of $\frac{\pi}{4}$.

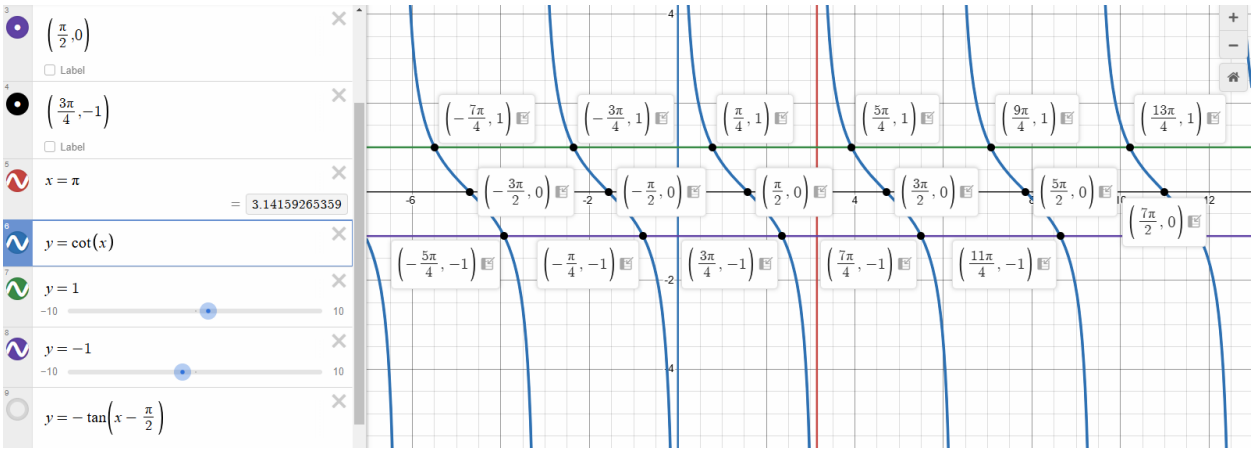
$$x=-\frac{\pi}{2}\ (VA),\left(-\frac{\pi}{4},-1\right),(0,0),\left(\frac{\pi}{4},1\right),x=\frac{\pi}{2}\ (VA)$$





Cotangent has vertical asymptotes at multiples of π, so our keypoints also need to go by $\frac{\pi}{4}$ (because the period is π not 2π), but we start at x=0.

$$x = 0, \left(\frac{\pi}{4}, 1\right), \left(\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{4}, -1\right), x = \pi$$



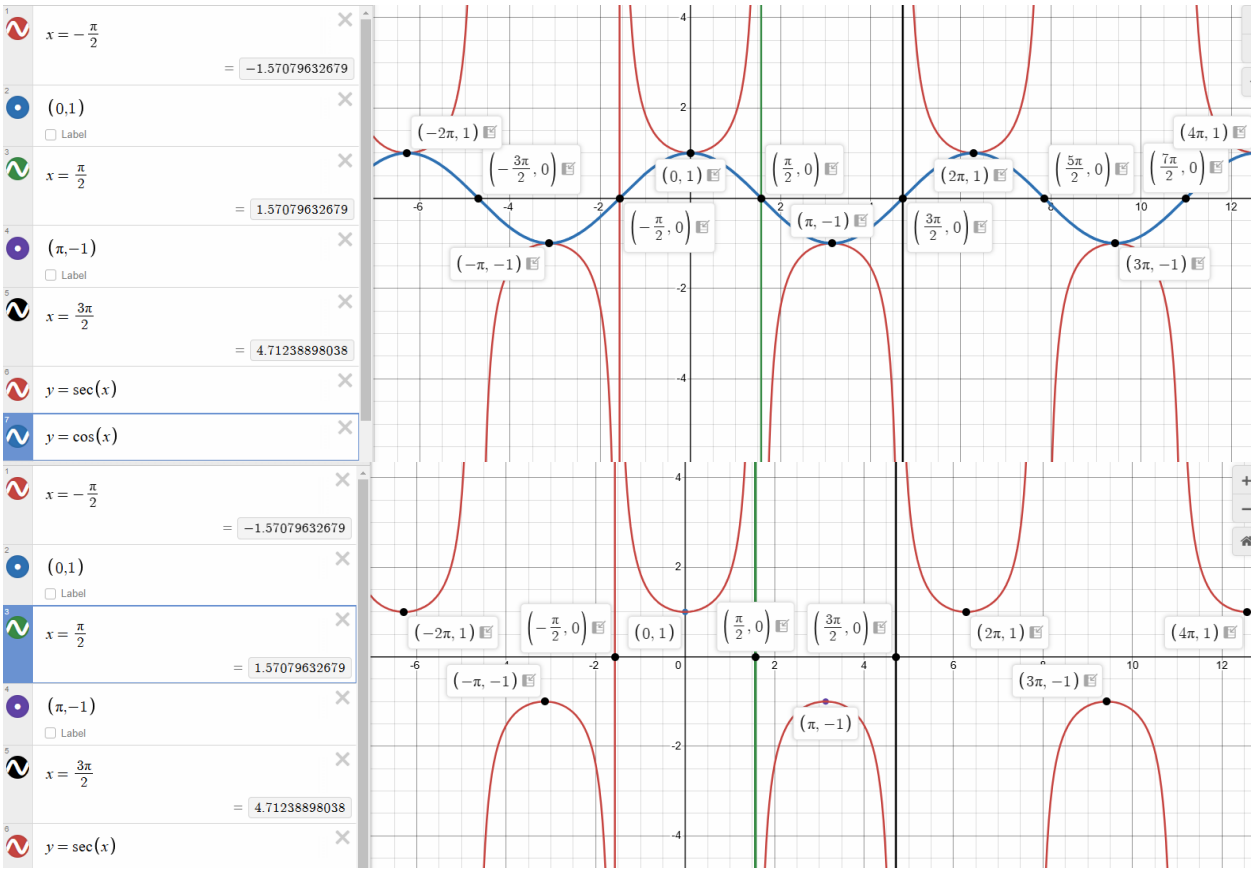
Secant and cosecant.

Period is 2π, means we need 3 vertical asymptotes to get the entire cycle.

Secant will have vertical asymptotes where tangent does (odd multiples of $\frac{\pi}{2}$), typically we plot between $-\frac{\pi}{2}$ and $\frac{3\pi}{2}$

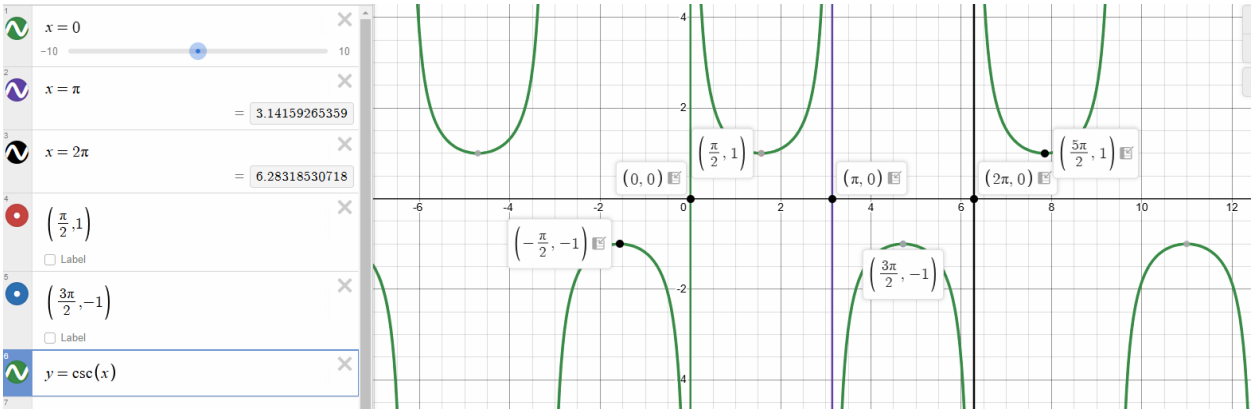
Keypoints:

$$x = -\frac{\pi}{2}\ (VA), (0,1), x = \frac{\pi}{2}\ (VA), (\pi, -1), x = \frac{3\pi}{2}\ (VA)$$



For cosecant, the asymptotes are at multiple of π.

$$x = 0\ (VA), \left(\frac{\pi}{2}, 1\right), x = \pi\ (VA), \left(\frac{3\pi}{2}, -1\right), x = 2\pi\ (VA)$$



We also apply the transformations to the domain and range to find the domain and range of a general trig function.

Your TI-84 calculator has sin(x), cos(x) and tan(x). Make sure you are in radian mode.
For cot(x) use 1/tan(x), for sec(x) use 1/cos(x) and for csc(x) use 1/sin(x).

$$\cos^{-1}(x) \neq \frac{1}{\cos(x)}$$

Next time, bring lots of questions for exam review, and we'll go over the procedures for taking the Exam on the 25th.