2/4/2025

Introduction to the course
Review of Functions, Inverses, Transformations

Functions:
Is a relation between two variables (x, the input variable and y, the output variable). In addition, a function relation has the property that every value for x results in only one value for y.

Examples of relations that are not functions:

$$x^2 + y^2 = 4$$
$$y^2 = x$$

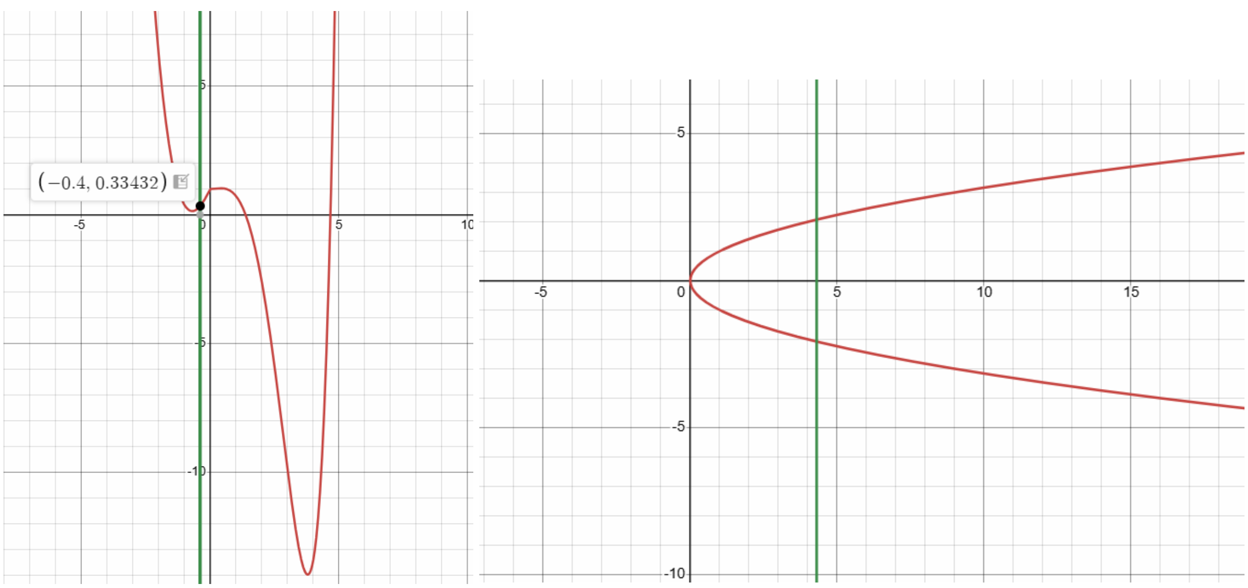
Example of functions:

$$x^2 = y$$
$$x^3 + |x| - e^x = y$$

To recognize things that are not functions look for y^2, $|y|$, etc. anything that results in x-symmetry
In general, if it's possible to solve your equation for y (with +/- signs), then it's a function.

I can also graph equations to see if they pass the vertical line test. If the vertical line cross the graph in more than one place, it's not a function.

https://www.desmos.com/calculator



Left: function, right: not a function (just a relation)

You can represent functions/relations in other forms, such tables or bubble graphs.
Still looking to see if I start out with any particular x, is there more than one y associated with it.

x	y
blue	Sally
green	Bobbie
brown	Joe
hazel	Ali
grey	Tabitha

In this configuration, this is a function.

x	y
blue	Sally
green	Bobbie
brown	Joe
hazel	Ali
grey	Tabitha

If I begin with someone with blue eyes, I don't know if I'm talking about Sally or Bobbie.
Just a relation and not a function.

Domain and Range of functions
The domain of a function is the possible values of x that can be put into the equation and get a real number out of the equation.

The range of the function is the possible values of y that the equation can produce (from values in its domain).

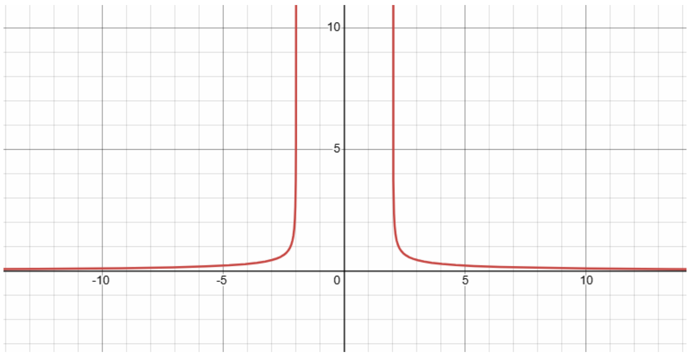
For example: $f(x) = e^x$, $g(x) = x^2$
The domain is all real numbers. For these the range of f is $(0, \infty)$, and the range of $[0, \infty)$.

$$h(x) = \frac{1}{\sqrt{x^2 - 4}}$$

Dividing: nothing in the denominator can make it zero (you can't divide by zero)
Square root: nothing under the square root can be negative.

$$x^2 - 4 > 0$$
$$x^2 > 4$$
$$x > 2, or\ x < -2$$

Domain: $(-\infty, -2) \cup (2, \infty)$



Based on the graph the range appears to be $(0, \infty)$

Function notation:

$$f(x) = x^2, f(2) =$$

Replace x in the function with 2,

$$f(2) = 2^2 = 4$$
$$(2,4)$$

Inverses:
The idea of an inverse is to start with the output and generate the input. Reversing the flow…

All relations have inverses. The question we have is are those inverses functions?

Inverse function notation: the inverse of the function f is $f^{-1}(x)$

This is **not** equal to $\frac{1}{f(x)}$!!!!

$$\frac{1}{f(x)} = [f(x)]^{-1}$$

If I have a table or a list of values, then I just switch the roles of x and y.
If I have an equation, then I switch x and y, and then try to solve for y.

$$f(x) = \frac{x}{x-2}$$

Find the inverse function (if it exists).

$$y = \frac{x}{x-2}$$

Switch x and y

$$x = \frac{y}{y-2}$$

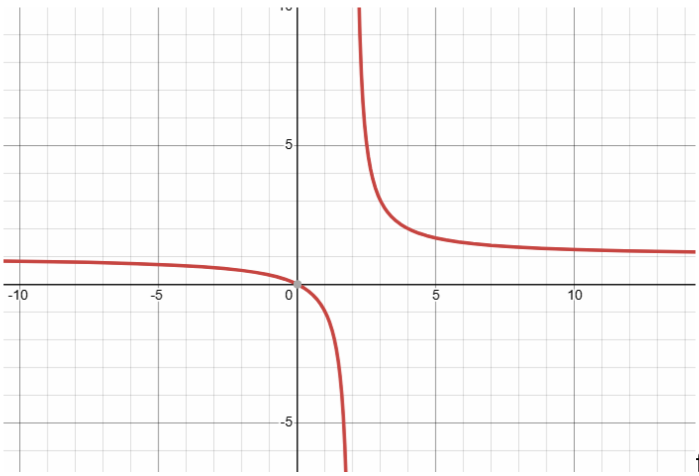
Try to solve for y.

$$x(y-2)=y$$
$$xy-2x=y$$
$$xy-y=2x$$
$$y(x-1)=2x$$
$$y=\frac{2x}{x-1}$$

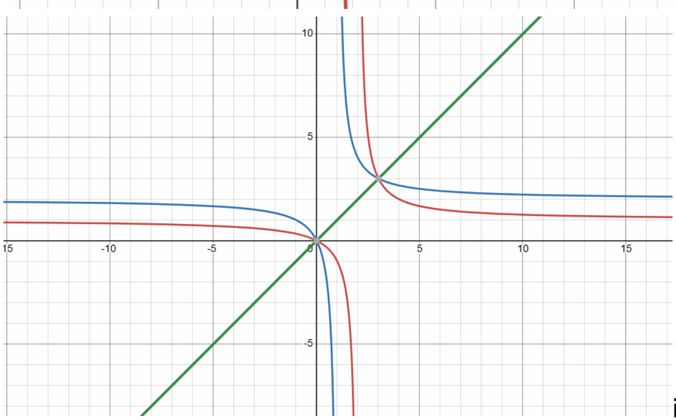
$$f^{-1}(x)=\frac{2x}{x-1}$$

For a function to have an inverse function, then it must be a function (each x produces one y), AND each y comes from only one x.

Visually, we use the horizontal line test (horizontal lines are y=constant), the function should only cross any horizontal line just one time.



f(x)



inverse function, symmetric to f across the line y=x

Both pass the vertical line test, and the horizontal line test.

Some functions don't pass the horizontal line test, such as $y=x^2$, to obtain an inverse function, you need to restrict the domain so that it will pass. For a parabola, we generally split it at the vertex.

The domain of the original function is the range of the inverse function, and the range of the original function is the domain of the inverse function. So if we can find an inverse function, that may help us t find the range of the original function algebraically.

Transformations.
We will pick this part up next time.