3/25/2025

Solving Trigonometric Equations

$$\sin(x) = \frac{1}{\sqrt{2}}$$

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = x$$

$$x = \frac{\pi}{4} + 2k\pi, \frac{3\pi}{4} + 2k\pi, k \in \mathbb{Z}$$

$$x = \left\{\dots, -\frac{15\pi}{4}, -\frac{7\pi}{4}, \frac{\pi}{4}, \frac{9\pi}{4}, \frac{17\pi}{4}, \dots\right\}, \left\{\dots, -\frac{13\pi}{4}, -\frac{5\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{4}, \frac{19\pi}{4}, \dots\right\}$$

$$\cos(x) = \frac{1}{2}$$

$$\cos^{-1}\left(\frac{1}{2}\right) = x$$

$$x = \frac{\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi$$

$$\sec(x) = 2$$

Is equivalent to $\cos(x) = \frac{1}{2}$

For the tangent and cotangent, they repeat every π, so you only have to find one angle, and then add $k\pi$.

$$\tan(x) = -1$$

$$\tan^{-1}(-1) = x$$

$$x = -\frac{\pi}{4} + k\pi$$

$$x = \left\{\dots, -\frac{5\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \dots\right\}$$

Example.

$$\sin(3x) = \frac{1}{2}$$

$$\sin(\alpha) = \frac{1}{2}$$

Find all the values of x in the $[0,2\pi)$

$$\alpha = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}$$

$$3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}$$

$$x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$$

$$\alpha = \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$$

$$x = \frac{\pi}{18} + \frac{2}{3}k\pi, \frac{5\pi}{18} + \frac{2}{3}k\pi$$

$$\cos(2x) = -\frac{1}{\sqrt{2}}$$

$$\cos(\alpha) = -\frac{1}{\sqrt{2}}$$

$$\alpha = \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}$$
$$2x = \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}$$

$$x = \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}$$

Example.

$$\sec(2x) = 1$$
$$\cos(2x) = 1$$
$$\alpha = 0,2\pi$$
$$2x = 0, 2\pi$$
$$x = 0, \pi$$

1 and -1 are exceptions for sine, cosine, secant and cosecant in that they only appear one time in the course of a full circle.
Every other possible value for these trig functions will give you two solutions.

$$\sin\left(\frac{x}{3}\right) = \frac{1}{\sqrt{2}}$$

$$\alpha = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$\frac{x}{3} = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$x = \frac{3\pi}{4}, \cancel{\frac{9\pi}{4}}$$

$$x = \frac{3\pi}{4}$$

Example.

$$\csc(x) = 0$$

There is no solution

Example.

$$2\cos\left(x + \frac{7\pi}{4}\right) = \sqrt{3}$$

$$\cos\left(x + \frac{7\pi}{4}\right) = \frac{\sqrt{3}}{2}$$

$$\cos(\alpha) = \frac{\sqrt{3}}{2}$$

$$\alpha = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$x + \frac{7\pi}{4} = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = -\frac{19}{12}\pi, -\frac{11}{12}\pi$$

$$x = \frac{5}{12}\pi, \frac{13}{12}\pi$$

Set the expression inside the trig function to be α. Then solve for the angles in one turn around the circle. Move any constant over. Verify that your angles are between 0 and 2π, and if not, add 2π until they are then. Then finish solving for x (multiplying the number of angles you need for the scalar as needed by adding 2π again before dividing).

Example.

$$\sec^2(x) = \frac{4}{3}$$

$$\cos^2(x) = \frac{3}{4}$$

$$\cos(x) = \pm\frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

Example.

$$\cos(x) = \sin(x)$$
$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$1 = \tan(x)$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

Example.

$$\sin(2x) = \sin(x)$$

$$2\sin(x)\cos(x) = \sin(x)$$

$$2\sin(x)\cos(x) - \sin(x) = 0$$
$$\sin(x)(2\cos x - 1) = 0$$

$$\sin(x) = 0$$
$$2\cos x - 1 = 0$$

$$\sin(x) = 0$$
$$x = 0, \pi$$
$$2\cos x - 1 = 0$$
$$2\cos x = 1$$
$$\cos(x) = \frac{1}{2}$$
$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$x = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$$

Example.

$$\cos(2x) = \sin(x)$$

$$1 - 2\sin^2 x = \sin(x)$$

$$0 = 2\sin^2 x + \sin(x) - 1$$

$$(2\sin x - 1)(\sin x + 1) = 0$$
$$2\sin x - 1 = 0$$
$$\sin(x) + 1 = 0$$

$$2\sin x - 1 = 0$$
$$2\sin x = 1$$
$$\sin x = \frac{1}{2}$$
$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\sin(x) + 1 = 0$$
$$\sin(x) = -1$$
$$x = \frac{3\pi}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

Example.

$$\tan^2 x = 1 - \sec(x)$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\sec^2 x - 1 = 1 - \sec(x)$$

$$\sec^2 x + \sec x - 2 = 0$$

$$(\sec x - 1)(\sec x + 2) = 0$$

$$\sec x - 1 = 0$$
$$\sec x = 1$$
$$x = 0$$

$$\sec x + 2 = 0$$
$$\sec x = -2$$
$$\cos x = -\frac{1}{2}$$
$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$$

Example.

$$\tan^3 x = 3 \tan x$$

$$\tan^3 x - 3 \tan x = 0$$

$$\tan(x)\,(\tan^2 x - 3) = 0$$

$$\tan(x) = 0$$
$$x = 0, \pi$$

$$\tan^2 x - 3 = 0$$
$$\tan^2 x = 3$$
$$\tan(x) = \pm\sqrt{3}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$x = 0, \pi, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$cot^4 x = 4\csc^2 x - 7$$

$$\cot^2(x) + 1 = \csc^2 x$$
$$\cot^4 x = 4(\cot^2 x + 1) - 7$$

$$\cot^4 x = 4\cot^2 x - 3$$
$$\cot^4 x - 4\cot^2 x + 3 = 0$$

$$u = \cot^2 x$$
$$u^2 - 4u + 3 = 0$$
$$(u - 3)(u - 1) = 0$$

$$(\cot^2 x - 3)(\cot^2 x - 1) = 0$$

$$\cot^2 x = 3$$
$$\cot(x) = \pm\sqrt{3}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\cot^2 x = 1$$
$$\cot x = \pm 1$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

When dealing with trig inequalities. Put everything on one side (if polynomial), set to 0. Factor. Solve for the break points where the sign changes. Then use a number line or a graph to determine which intervals satisfy the inequality and which don't.

This is the end of the material for Exam #2.