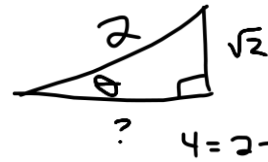
3/4/2025

Finish working with Inverses Trig Functions
Application problems

Section 10.6

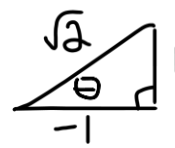
7. $\arcsin\left(\frac{\sqrt{2}}{2}\right)$



$4=2+b^2$
$2=b^2$
$b=\sqrt{2}$

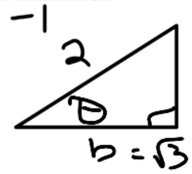
$$\sin(\theta)=\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}$$
$$\theta=\frac{\pi}{4}$$

27. $\operatorname{arccot}(-1)$



$$\cot(\theta)=-1$$
$$\theta=\frac{3\pi}{4}$$

34. $\operatorname{arccsc}(2)$



$$\csc(\theta)=2, \sin(\theta)=\frac{1}{2}$$
$$\theta=\frac{\pi}{6}$$

59. $\sin\left(\arcsin\left(\frac{3}{5}\right)\right)$

The domain of arcsine is [-1,1]. 3/5 is in this interval, so this is defined.

$$\sin(\theta)=\frac{3}{5}\rightarrow \arcsin\left(\frac{3}{5}\right)=\theta$$
$$\sin(\theta)=?=\frac{3}{5}$$

In this case, the sine and the arcsine do cancel each other out.

71. $\tan(\arctan(3\pi))$

$$\arctan(3\pi)=\theta\rightarrow \tan(\theta)=3\pi$$

The domain of arctangent is all real numbers (since the range of tangent is all real numbers)

$$\tan(\theta)=3\pi$$

Here, the tangent and its inverse canceled.

80. $\sec(\operatorname{arcsec}(0.75))$

$$\operatorname{arcsec}(0.75)=\theta$$
$$\sec(\theta)=0.75=\frac{3}{4}\rightarrow \cos(\theta)=\frac{4}{3}$$

There is no angle where the cosine value is bigger than 1, and there is no angle where the secant has a value between (-1,1).

These cannot cancel because the inside expression is not defined.

89. $\arcsin\left(\sin\left(\frac{3\pi}{4}\right)\right)$

$$\sin\left(\frac{3\pi}{4}\right) = ? = \frac{1}{\sqrt{2}}$$

In the second quadrant, the sine is positive.

$$\arcsin\left(\frac{1}{\sqrt{2}}\right) = ? = \frac{\pi}{4}$$

The range of the arcsine is only in the first and fourth quadrants.

95. $\arccos\left(\cos\left(-\frac{\pi}{6}\right)\right)$

$$\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$
$$\arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

99. $\arctan(\tan(\pi))$

$$\tan(\pi) = 0$$
$$\arctan(0) = 0$$

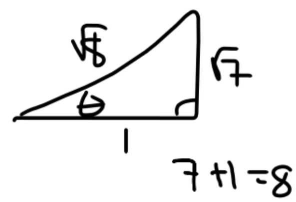
111. $\operatorname{arcsec}\left(\sec\left(\frac{5\pi}{3}\right)\right)$

$$\sec\left(\frac{5\pi}{3}\right) = 2$$

In the fourth quadrant, cosine is positive

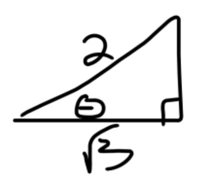
$$\operatorname{arcsec}(2) = \frac{\pi}{3}$$

137. $\cos\left(\arctan\left(\sqrt{7}\right)\right)$ $\frac{\sqrt{7}}{1}$



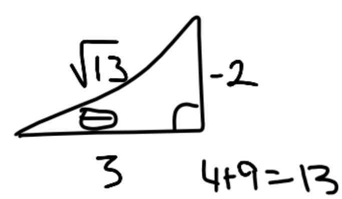
$$\arctan(\sqrt{7}) = \theta$$
$$\cos(\theta) = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}} = \cdots$$

148. $\sec\left(\arccos\left(\frac{\sqrt{3}}{2}\right)\right)$



$$\cos\left(\frac{\sqrt{3}}{2}\right) = \theta$$
$$\sec(\theta) = \frac{2}{\sqrt{3}}$$

154. $\csc\left(\arctan\left(-\frac{2}{3}\right)\right)$

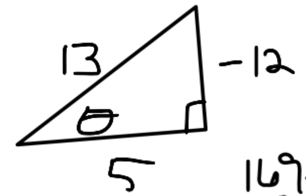


$$\arctan\left(-\frac{2}{3}\right) = \theta$$

$$\csc(\theta) = \frac{\sqrt{13}}{-2} = -\frac{\sqrt{13}}{2}$$

149. $\sec\left(\arcsin\left(-\frac{12}{13}\right)\right)$

$$\arcsin\left(-\frac{12}{13}\right) = \theta$$

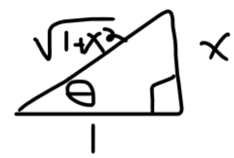


$169 - 144 = 25$

$\sec(\theta) = \frac{13}{5}$

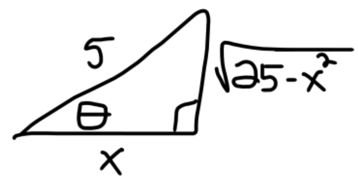
166. $\cos(\arctan(x))$

$$\arctan(x) = \theta = \arctan\left(\frac{x}{1}\right)$$
$$\cos(\theta) = \frac{1}{\sqrt{1+x^2}}$$



174. $\sin\left(\arccos\left(\frac{x}{5}\right)\right)$

$$\arccos\left(\frac{x}{5}\right) = \theta$$
$$\sin(\theta) = \frac{\sqrt{25 - x^2}}{5}$$



$25 = x^2 + a^2$

In these simple equations, we have a trig function value, and we want to find the value of the angle.

188. $\sin(x) = \frac{7}{11}$

Use our calculator find the value of the angle.

$$\sin^{-1}\left(\frac{7}{11}\right) = 39.52°, 0.68977\ radians$$

198. $\tan(x) = -\sqrt{10}$

$\tan^{-1}(-\sqrt{10}) = -1.2645 \ldots radians, -72.45°$ (stop here for just the equation, with no context)
Get this back into the second quadrant is add π (or $180°$).
$\theta = 107.54°, 1.8771\ radians$ (stop here for a word problem where you need an obtuse angle)

196. $\sec(x) = \frac{3}{2}$

$$\cos(x) = \frac{2}{3}$$
$$\cos^{-1}\left(\frac{2}{3}\right) = 0.8410 \ldots radians, 48.19°$$

223. $f(x) = \arccos\left(\frac{3x-1}{2}\right)$

Find the domain and range of the function.
State the domain of the arccos(x)
Start with $[-1,1]$

$$-1 \le x \le 1$$

Replace x with the stuff inside the function and then solve:

$$-1 \le \frac{3x-1}{2} \le 1$$

$$-2 \le 3x - 1 \le 2$$
$$-1 \le 3x \le 3$$

$$-\frac{1}{3} \le x \le 1$$

Domain is now $\left[-\frac{1}{3}, 1\right]$
Range is unchanged $[0,2\pi]$

In most applications for now, you'll be working with right triangles. In previous sections, we had the angle and one side and could use trig functions to find another side. If we are looking for the angle from two sides, we need inverse trig functions.

211. A guy wire 1000 feet long is attached to the top of a tower. When pulled taut it touches level ground 360 feet from the base of the tower. What angle does the wire make with the ground? Express your answer using degree measure rounded to one decimal place. θ

$\cos(\theta) = \frac{360}{1000} = \frac{36}{100} = 0.36$

$\theta = 68.9°$