3/6/2025

Verifying Trig Identities (examples taken from 10.3 in the Stitz book)
Proof techniques

Example.
Verify the identity.

$$\csc(\theta)\cos(\theta) = \cot(\theta)$$

$$\frac{1}{\sin(\theta)}\cos(\theta) = \frac{\cos(\theta)}{\sin(\theta)} = \cot(\theta)$$

Q.E.D. = quod erat demonstrandum (thus it has been shown)

$$\frac{\sin(\theta)}{\cos^2(\theta)} = \sec(\theta)\tan(\theta)$$

$$\sec(\theta)\tan(\theta) = \frac{1}{\cos(\theta)}\left(\frac{\sin(\theta)}{\cos(\theta)}\right) = \frac{\sin(\theta)}{\cos^2(\theta)}$$

QED

$$\frac{1-\cos(\theta)}{\sin(\theta)} = \csc(\theta) - \cot(\theta)$$

$$\frac{1}{\sin(\theta)} - \frac{\cos(\theta)}{\sin(\theta)} = \csc(\theta) - \cot(\theta)$$

QED.

$$\frac{\sec(\theta)}{1+\tan^2(\theta)} = \cos(\theta)$$

$$\cos^2(\theta) + \sin^2(\theta) = 1$$
$$\tan^2(\theta) + 1 = \sec^2(\theta)$$
$$\cot^2(\theta) + 1 = \csc^2(\theta)$$

$$\frac{\sec(\theta)}{\sec^2\theta} = \frac{1}{\sec(\theta)} = \cos(\theta)$$

QED

In a formal proof, we would want to narrate or explain every step.

$$\tan^3\theta = \tan(\theta)\sec^2\theta - \tan(\theta)$$

$$\tan(\theta)\sec^2\theta - \tan(\theta) = \tan(\theta)\left[\sec^2\theta - 1\right] = \tan\theta\left[\tan^2\theta\right] = \tan^3\theta$$

QED

$$\frac{1-\tan\theta}{1+\tan\theta}=\frac{\cos(\theta)-\sin(\theta)}{\cos\theta+\sin\theta}$$

$$\frac{1-\tan\theta}{1+\tan\theta}=\frac{1-\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin\theta}{\cos\theta}}=\frac{\left(1-\frac{\sin\theta}{\cos\theta}\right)\cos\theta}{\left(1+\frac{\sin\theta}{\cos\theta}\right)\cos\theta}=\frac{\cos\theta-\frac{\sin\theta\cancel{\cos\theta}}{\cancel{\cos\theta}}}{\cos\theta+\frac{\sin\theta\cancel{\cos\theta}}{\cancel{\cos\theta}}}=\frac{\cos(\theta)-\sin(\theta)}{\cos\theta+\sin\theta}$$

QED

$$\frac{1}{1-\cos\theta}+\frac{1}{1+\cos\theta}=2\csc^2\theta$$

Find the common denominator

$$\frac{1}{1-\cos\theta}\left(\frac{(1+\cos\theta)}{1+\cos\theta}\right)+\frac{1}{1+\cos\theta}\left(\frac{(1-\cos\theta)}{1-\cos\theta}\right)=\frac{(1+\cos\theta)}{1-\cos^2\theta}+\frac{(1-\cos\theta)}{1-\cos^2\theta}=$$

$$\frac{(1+\cos\theta+1-\cos\theta)}{\sin^2\theta}=\frac{2}{\sin^2\theta}=2\csc^2\theta$$

QED.

$$\frac{1}{1+\sin\theta}=\sec^2\theta-\sec\theta\tan\theta$$

$$\sec^2\theta-\sec\theta\tan\theta=\frac{1}{\cos^2\theta}-\frac{1}{\cos\theta}\left(\frac{\sin\theta}{\cos\theta}\right)=\frac{1}{\cos^2\theta}-\frac{\sin\theta}{\cos^2\theta}=\frac{1-\sin\theta}{\cos^2\theta}=\frac{1-\sin\theta}{1-\sin^2\theta}=$$

$$\frac{\cancel{(1-\sin\theta)}}{\cancel{(1-\sin\theta)}(1+\sin\theta)}=\frac{1}{1+\sin\theta}$$

QED

$$\frac{1}{\csc\theta+1}+\frac{1}{\csc\theta-1}=2\sec\theta\tan\theta$$

$\frac{1}{\csc\theta+1}+\frac{1}{\csc\theta-1}$	$2\sec\theta\tan\theta$
$\frac{1}{\csc\theta+1}\left(\frac{\csc\theta-1}{\csc\theta-1}\right)+\frac{1}{\csc\theta-1}\left(\frac{\csc\theta+1}{\csc\theta+1}\right)$	$2\left(\frac{1}{\cos\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right)$
$\frac{\csc\theta-1}{\csc^2\theta-1}+\frac{\csc\theta+1}{\csc^2\theta-1}$	$\frac{2\sin\theta}{\cos^2\theta}$

$\dfrac{\csc\theta - 1 + \csc\theta + 1}{\csc^2\theta - 1}$	$\dfrac{2\sin\theta}{1-\sin^2\theta}$
$\dfrac{2\csc\theta}{\csc^2\theta - 1}$	
$\dfrac{2\left(\frac{1}{\sin\theta}\right)}{\left(\frac{1}{\sin\theta}\right)^2 - 1} = \dfrac{\frac{2}{\sin\theta}}{\frac{1}{\sin^2\theta} - 1}$	
$\dfrac{\frac{2}{\sin\theta}}{\frac{1}{\sin^2\theta} - 1}\left(\dfrac{\sin^2\theta}{\sin^2\theta}\right) = \dfrac{\frac{2\sin^2\theta}{\sin\theta}}{\frac{\sin^2\theta}{\sin^2\theta} - \sin^2\theta}$ =1	
$\dfrac{2\sin\theta}{1-\sin^2\theta}$	