4/24/2025

Parametric Equations
Review for Exam #3

Parametric Equation grapher:
https://www.desmos.com/calculator/rfj91yrxob
https://www.geogebra.org/m/cAsHbXEU

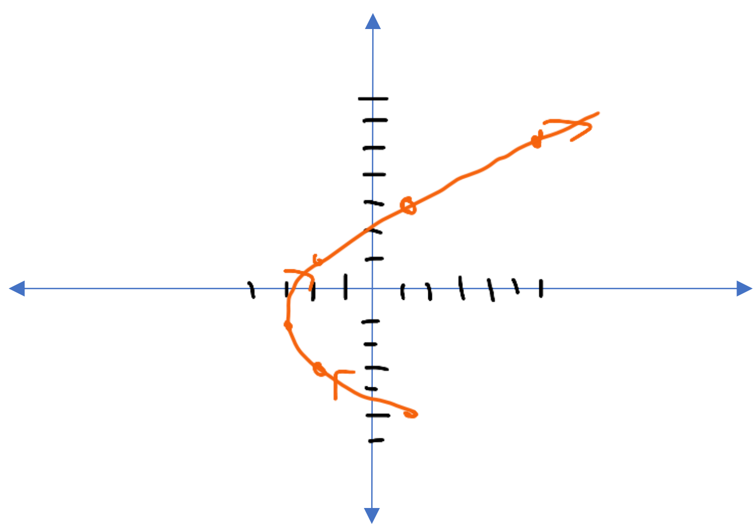
We want to describe the position of x and y in terms of a third variable (time). For instance, we have a particle traveling along a path defined by the function (or relation), we want to be able to describe that motion along the path. The time variable can alter the portion of the curve that is covered, or the direction in which that curve is traversed.

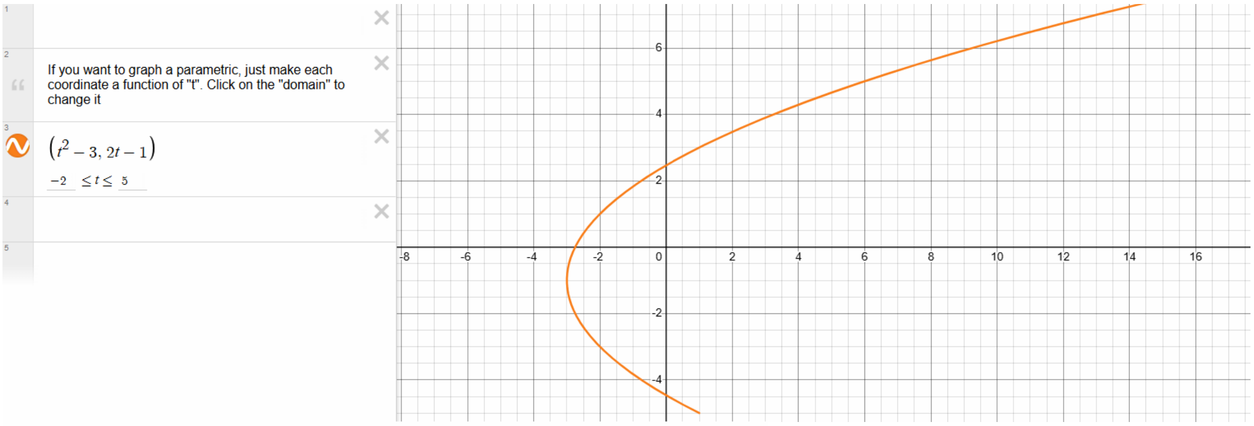
The third variable is called the parameter, and usually t.

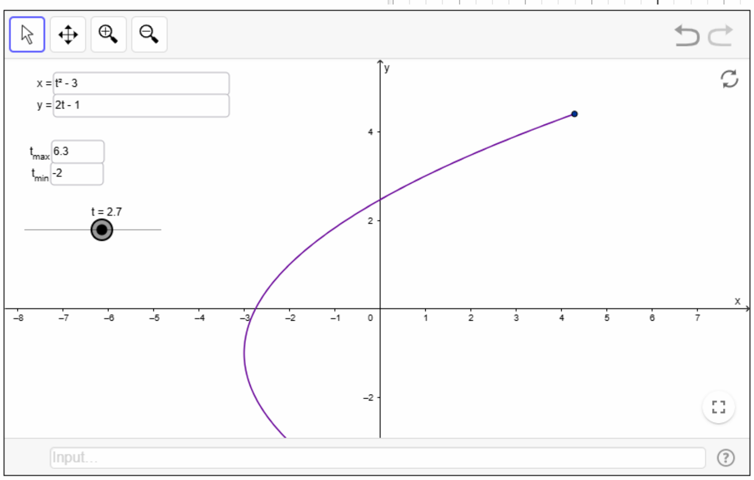
A set of equations where $x = x(t), y = y(t)$.
There is an orientation to the curve (since time only goes in one direction). We have to mark this on the graph with an arrow pointed along the curve in the direction in which t is increasing.

Example. Plot the curved defined by the parametric equations $x = t^2 - 3, y = 2t - 1$. For $t \geq -2$.

t	$x(t)$	$y(t)$
-2	1	-5
-1	-2	-3
0	-3	-1
1	-2	1
2	1	3
3	6	5







If I want to put this parametric form back into standard function form, then solve for t in one of the equations (or some common function of t), and then plug in to the other equation.

$$y = 2t - 1$$

$$t = \frac{y+1}{2}$$

$$x = t^2 - 3$$

$$x = \left(\frac{y+1}{2}\right)^2 - 3$$

$$x(y) = \frac{1}{4}(y+1)^2 - 3$$

Solve for y if you can, but don't introduce $\pm$ signs.

Example. Convert the parametric functions into rectangular coordinates (x,y) only.

$$x = e^{-t}, y = e^{-2t}$$

$$x = e^{-t}, y = (e^{-t})^2$$

$$y = x^2$$

The long way:

$$x = e^{-t}$$
$$\ln x = -t$$
$$-\ln x = t$$

$$y = e^{-2(-\ln x)} = e^{2\ln x} = e^{\ln x^2} = x^2$$

I do end up at the same place, but solving for t made the process longer (on both ends).

Converting from rectangular form (x,y) to parametric form.

Typically, if your function is explicit (as in $y = f(x)$), then you can just let $x = t$, and then substitute to get $y(t)$.

Example.

$$y = 3x^2 + x - 1$$

In parametric form:

$$x = t, y = 3t^2 + t - 1$$

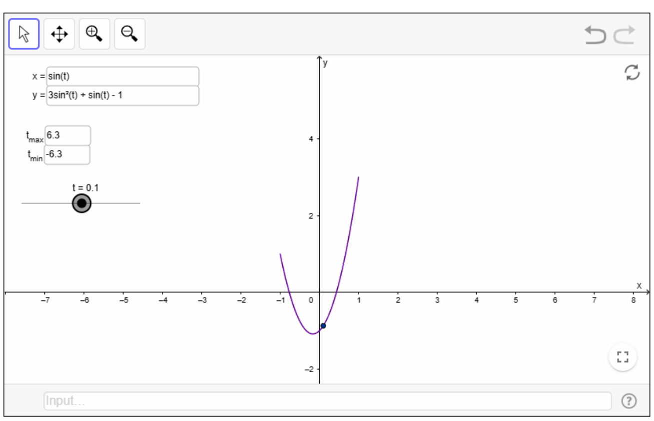
However, you could choose other parameters.
Suppose you let $x = 2t$
Then $y = 3(2t)^2 + 2t - 1 = 12t^2 + 2t - 1$

Suppose you let $x = e^t$

$$y = 3e^{2t} + e^t - 1$$

Suppose you let $x = \sin(t)$

$$y = 3\sin^2 t + \sin t - 1$$



If you flip the sign of t, (use -t) it switches the orientation of the curve.

Some curves can be expressed in parametric form (as if they functions in x and y, but not as a function y in terms of x).

A circle is not a function of y in terms of x.

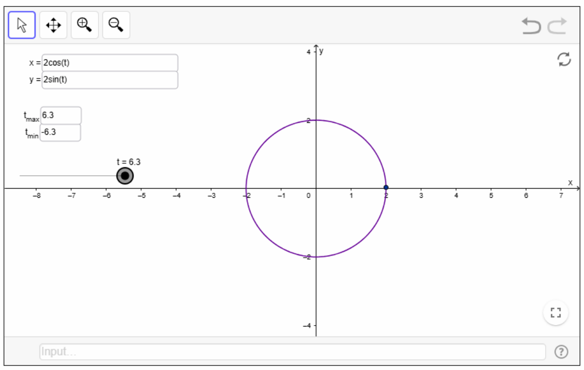
$$x^2 + y^2 = 4$$

Can convert to parametric form using polar coordinates relationships:

$$x = r\cos t\,, y = r\sin t$$

A circle of radius 2:

$$x = 2\cos t\,, y = 2\sin t$$



orientation is counterclockwise

If you switch the sin(t) and cos(t), the starting point will not be the positive x-axis, it will be the positive y-axis, and the orientation will be clockwise.

I can get any circle I want, with any radius and any center:

$$x(t) = r\cos(t) + h$$
$$y(t) = r\sin(t) + k$$

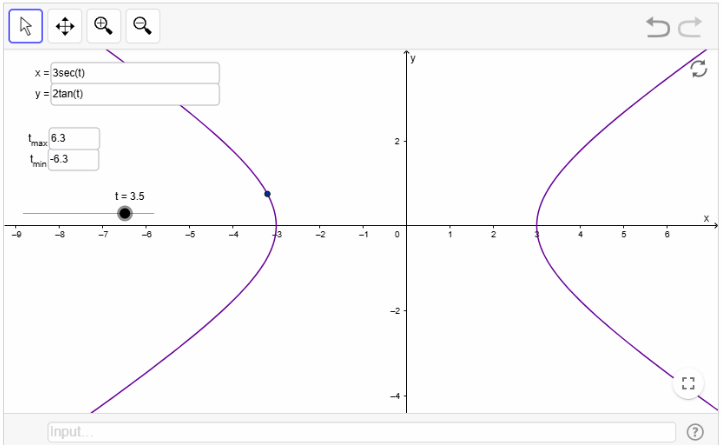
If I adjust the t to -t, then I'll go backwards around the circle. If I replace t with 2t, I'll go around the circle faster.

To do an ellipse, it's basically the same except that the radius is not the same value in both coordinates.

$$x(t) = a\cos(t) + h$$
$$y(t) = b\sin(t) + k$$

To obtain a hyperbola, instead of sine and cosine, use tangent and secant.

Transverse-x orientation, x=a sec(t), y= b tan(t)



And transverse-y (opening up and down) is the reverse: x=a tan(t), y= b sec(t).

(You can also do this with hyperbolic trig functions… you'll likely see these in calc 2 if you haven't already).

Be able to plot a parametric curve in parametric form. Indicate the orientation of the curve on the graph. Be able to convert parametric equations back to rectangular form, and convert rectangular form into parametric form. Recognize and apply comic conic forms in parametric equations. Be able to draw by hand, but you can check your work with technology.

Here ends Exam #3 material.

Laws of Sines and Cosines
Polar Coordinates, Graphing Polar Curves
Complex Numbers in Polar form: multiplying, dividing, rooting, and raising a largish power
Conics: Circle, Ellipse, Parabola, Hyperbola; Rectangular and Polar Form
Vectors: adding/subtracting, plotting, dot product, angle between, projection
Parametric equations: switching forms, plotting, special curves

DeMoivre's Theorem is for raising complex numbers to a power.

Convert to polar form. Raise the magnitude to the power, and then multiple the angle by the power.

$$z = r\,e^{i\theta}$$

$$z^n = r^n e^{in\theta} = r^n(\cos(n\theta) + i\sin(n\theta))$$

Roots: the nth root has n roots…

$$if\ z = r\,e^{i\theta}$$

Then $\sqrt[n]{z}$ (all of them) are:

$$z_1 = \sqrt[n]{r}e^{\frac{i\theta}{n}}$$
$$z_2 = \sqrt[n]{r}e^{\frac{i\theta+2\pi}{n}}$$
$$z_3 = \sqrt[n]{r}e^{\frac{i\theta+4\pi}{n}}$$

$$z_4 = \sqrt[n]{r}e^{\frac{i\theta+6\pi}{n}}$$
$$z_5 = \sqrt[n]{r}e^{\frac{i\theta+8\pi}{n}}$$
$$\ldots$$

Keep doing this until you have n roots (if you go to n+1 roots, you'll get the same value as the first root, but in a different form).

P in a parabola is the distance from the vertex to the focus. Put 4p in the equation on the side of the linear term.