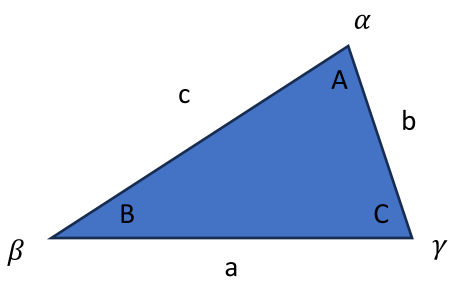
4/3/2025

Law of Sines
Law of Cosines



Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

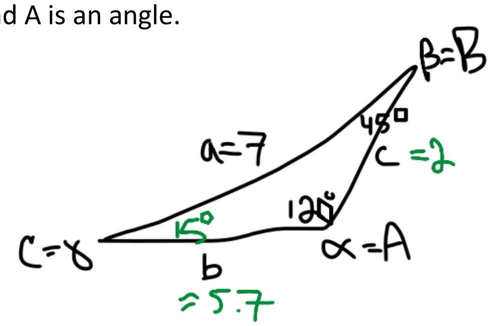
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosine:

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$b^2 = c^2 + a^2 - 2ac\cos B$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$

Law of Sines is used when we have SSA info or SAA info… S is a side, and A is an angle.

Law of Cosines: SAS or SSS



1. $\alpha = 120°$, $a = 7$ units, $\beta = 45°$

Law of Sines because I have angle A (α) and side a. (SAA)

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

$$\frac{7}{\sin(120°)} = \frac{b}{\sin(45°)}$$

$$\frac{7\sin(45°)}{\sin(120°)} = b$$

$$b \approx 5.71547\ldots$$

$$C = \gamma = 180 - 120 - 45 = 15°$$

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$\frac{7}{\sin(120°)} = \frac{c}{\sin(15°)}$$

$$\frac{7\sin(15°)}{\sin(120°)} = c$$

$$c \approx 2.09200 \ldots$$

SAA triangles are the easiest. You can get the missing angle from the fact that all angles add to 180-degrees. And then use law of sines to get the two missing sides. There is only one possible outcome.

SSA triangle can produce three possible outcomes:

1) There is just one triangle.
2) There are two possible triangles
3) There are no possible triangles.

No possible triangles case, what we end up with mathematically is sin(angle)>1.

In other cases, we always have to look for the second possible triangle. Typically, this will happen in SSA problem (one angle given) and the angle is relatively small.
Find the second angle using the law of sines (in the first quadrant). Convert that angle to the second quadrant (using a reference angle). Check to see if the angle you were given and the obtuse angle you just found is less than 180-degrees. If yes, there are two triangle solutions to the given information. If not, there is only one triangle using the first (acute) angle you found.

3. $\alpha = 30°$, $a = 1$ units, $c = 4$ units

5. $\alpha = 30°$, $a = 3$ units, $c = 4$ units

$$\frac{\sin(30°)}{1} = \frac{\sin(\gamma)}{4}$$

$$\sin(\gamma) = 4\sin(30°) = 2$$

$$\sin(\gamma) = 2$$
$$\sin^{-1}(2) = \gamma$$

No solution, since no angles can produce a ratio of sides bigger than 1.

$$\frac{\sin(30°)}{3} = \frac{\sin(\gamma)}{4}$$

$$\frac{4\sin(30°)}{3} = \sin(\gamma)$$

$$\sin(\gamma) = \frac{2}{3}$$

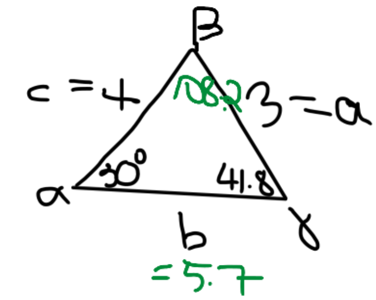
$$\sin^{-1}\left(\frac{2}{3}\right) = 41.8103\ldots°$$

$$\beta = 180 - 30 - 41.8 = 108.2°$$

$$\frac{3}{\sin 30°} = \frac{b}{\sin(108.2°)}$$

$$b = \frac{3\sin(108.2°)}{\sin 30°}$$

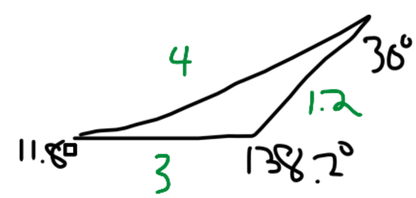
$$b \approx 5.6998\ldots.$$



Triangle 1: solved

Is there a triangle 2?
Our acute angle that we found from the law of sines was 41.8°.
There is an acute angle with same sine value: $180° - 41.8° = 138.2°$

If α is 30° and γ is 138.2°, is there enough room in the triangle for angle β?



$$\alpha + \gamma + \beta = 180°$$

$$180 - 138.2 - 30 = 11.8°$$

$$\beta = 11.8°$$

$$\frac{3}{\sin 30°} = \frac{b}{\sin(11.8°)}$$

$$\frac{3\sin(11.8°)}{\sin(30°)} = b$$

$$b \approx 1.22697\ldots.$$

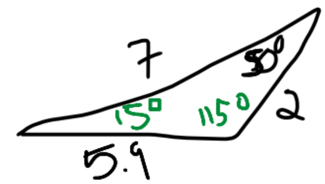
Law of Cosines

1. $\beta = 50°$, $a = 7$ units, $c = 2$ units

2. $a = 4$ units, $b = 7$ units, $c = 5$ units

SAS = the angle we have is opposite the side we are missing.

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$



$$b^2 = (7)^2 + (2)^2 - 2(7)(2)\cos(50°)$$
$$b^2 = 35.0019\ldots$$

$$b \approx 5.916244\ldots$$

$$\frac{\sin(50°)}{5.916244\ldots} = \frac{\sin(\alpha)}{7}$$

$$\frac{7\sin(50°)}{5.916244\ldots} = \sin(\alpha)$$

$$\sin(\alpha) \approx 0.90637\ldots$$

$$\alpha \approx 65.0085\ldots°$$

But this is acute, and we know from the shape of the triangle that it has to be obtuse:

$$180 - 65 = 115°$$

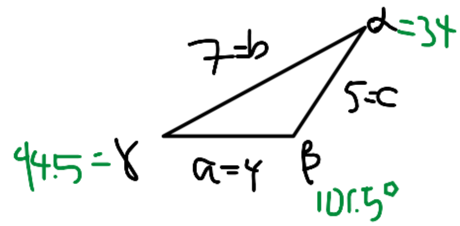
$$180 - 50 - 115 = 15$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$\frac{a^2 - b^2 - c^2}{-2bc} = \cos(\alpha)$$

$$\frac{7^2 - 35 - 2^2}{-2(5.9)(2)} = \cos(\alpha)$$

$$\cos(\alpha) \approx -0.4237\ldots$$

$$\alpha \approx 115°$$



SSS case, we use all the sides to find the first angle.

$$b^2 = c^2 + a^2 - 2ac\cos\beta$$

$$\frac{(7^2 - 5^2 - 4^2)}{-2(4)(5)} = \cos\beta$$

$$\cos\beta = -0.2$$

$$\beta = 101.5\ldots°$$

$$\frac{\sin(\alpha)}{4} = \frac{\sin(101.5°)}{7}$$

$$\sin(\alpha) = \frac{4\sin(101.5°)}{7} \approx 0.55995\ldots$$

$$\alpha \approx 34°$$

$$180 - 101.5 - 34 = 44.5°$$