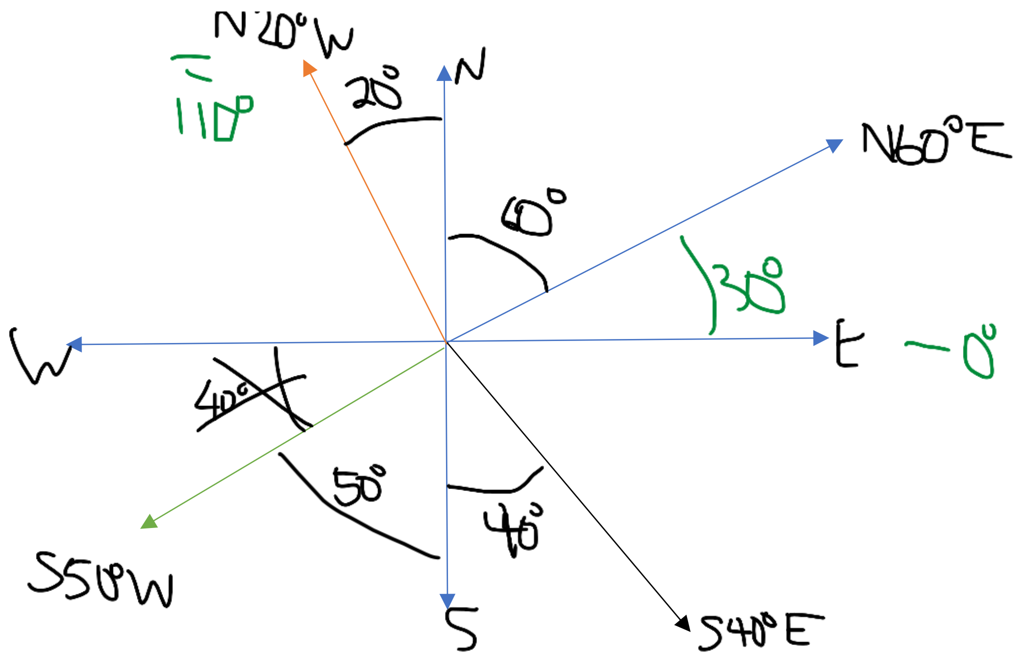
4/8/2025

Bearings
Polar Coordinates
Graphs in Polar Coordinates

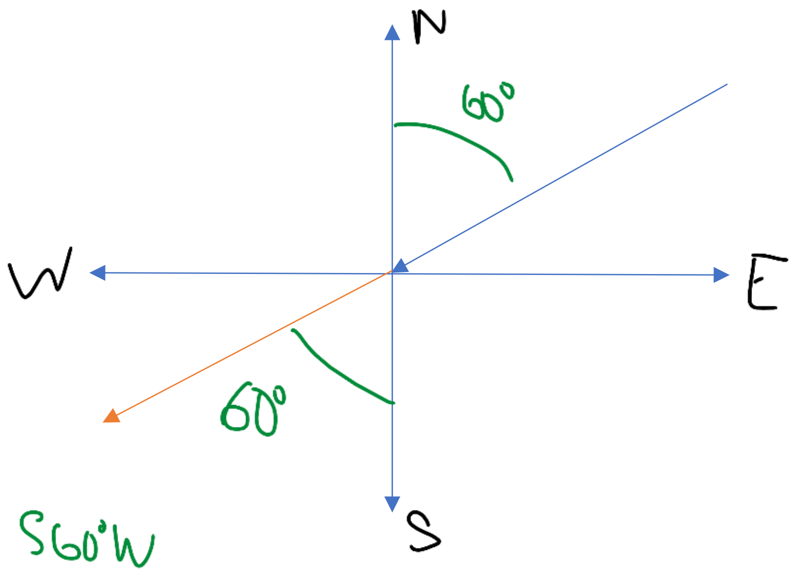
Bearings



Bearings are angles measured always from the North or South direction. Measure the angular distance from north or south and tack on which side of north or south you are on… are you on the east side of north-south, or on the west side of north-south.

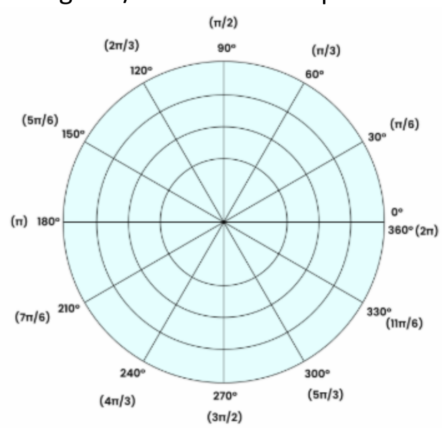
$N60°E$ is a 60-degree angle measured from the north, on the side of the east direction (on the right side).

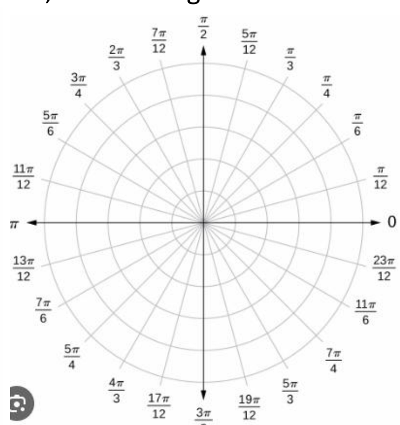
Wind direction has one additional complication, and that is that wind is measured in the direction the wind is coming FROM, not the direction it's going in. You need the direction its going in to do the problem.



A wind is coming from $N60°E$, the direction its going IN is… $S60°W$

Polar Coordinates
Polar coordinates are measured in magnitude and direction.
O-degrees/0 radians is the positive x-axis, and the angles increase counterclockwise.





Polar coordinates (r, θ)

Conversion formulas:

$$x^2 + y^2 = r^2$$
$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$\tan^{-1}\left(\frac{y}{x}\right) = \theta$$

Converting points

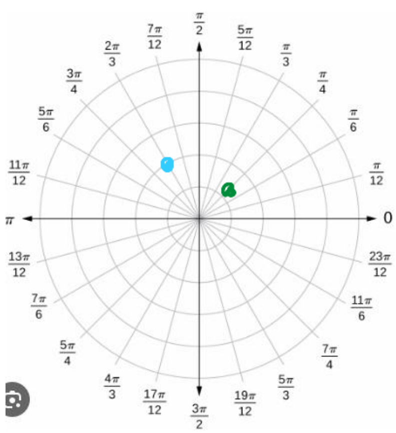
From rectangular coordinate (a point) to polar coordinates (a point), we'll use the $x^2 + y^2 = r^2$, $\tan^{-1}\left(\frac{y}{x}\right) = \theta$.

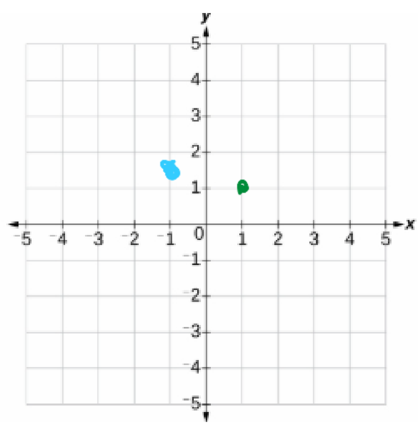
Consider the point (1,1)
Convert the point in rectangular coordinates to a point in polar coordinates.

$$x = 1, y = 1$$
$$(1)^2 + (1)^2 = 1 + 1 = 2$$
$$r^2 = 2$$
$$r = \sqrt{2}$$

$$\tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$$

Our point in polar coordinates is $\left(\sqrt{2}, \frac{\pi}{4}\right)$





In rectangular coordinates every point has a unique pair of values that mark its location on the plane.
In polar coordinates, every point has multiple representations (not unique).

The positive radius and angle between 0 and 2π to be the standard expression of a point.

$$\left(\sqrt{2}, \frac{\pi}{4}\right) = \left(\sqrt{2}, \frac{9\pi}{4}\right) = \left(\sqrt{2}, -\frac{7\pi}{4}\right) = \left(-\sqrt{2}, \frac{5\pi}{4}\right) = \left(-\sqrt{2}, -\frac{3\pi}{4}\right)$$

Convert the point $\left(-1, \sqrt{3}\right)$ in rectangular coordinates to a point in polar coordinates.

$$x^2 + y^2 = r^2$$
$$(-1)^2 + \left(\sqrt{3}\right)^2 = 1 + 3 = 4$$
$$r^2 = 4$$
$$r = 2$$

$$\tan^{-1}\left(\frac{\sqrt{3}}{-1}\right) = -\frac{\pi}{3}$$

Move this angle into the second quadrant by adding π

$$\theta = \frac{2\pi}{3}$$

Point is $\left(2, \frac{2\pi}{3}\right)$

Alternatively, you can use the negative radius to move to the other side of the graph…

$$\left(-2, -\frac{\pi}{3}\right)$$

Convert the point $\left(3, \frac{\pi}{6}\right)$ to rectangular coordinates.

$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$x = 3\cos\left(\frac{\pi}{6}\right) = 3\left(\frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{3}}{2}$$
$$y = 3\sin\left(\frac{\pi}{6}\right) = 3\left(\frac{1}{2}\right) = \frac{3}{2}$$

$$\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$$

Now we want to convert formulas/functions.

Lines:

$$x = 3$$

Convert to polar
Get rid of all x and y in the equation, constants stay the same, and r is the function variable (so solve for r if possible).

$$r\cos\theta = 3$$
$$r = \frac{3}{\cos\theta} = 3\sec\theta$$

A vertical line in polar coordinates is a scalar of secant.

$$y = 6$$
$$r\sin\theta = 6$$
$$r = 6\csc\theta$$

A horizontal line in polar coordinates is a scalar of cosecant.

A line through the origin?

$$y = x, y = \sqrt{3}x$$

$$y = x$$
$$r\sin\theta = r\cos\theta$$

$$\sin\theta = \cos\theta$$
$$\theta = \frac{\pi}{4}$$

$$\tan\theta = 1$$

$$r\sin\theta = \sqrt{3}r\cos\theta$$
$$\sin\theta = \sqrt{3}\cos\theta$$

$$\tan\theta = \sqrt{3}$$
$$\theta = \frac{\pi}{3}$$

Any line through the origin is expressed as an angle…

$$\theta = \frac{5\pi}{6}$$

Write this in rectangular coordinates.

$$\tan^{-1}\left(\frac{y}{x}\right) = \theta$$

$$\tan(\theta) = \frac{y}{x}$$

$$\tan\left(\frac{5\pi}{6}\right) = -\frac{1}{\sqrt{3}}$$

$$\frac{y}{x} = -\frac{1}{\sqrt{3}}$$

$$y = -\frac{1}{\sqrt{3}}x$$

Circles:

$$x^2 + y^2 = 25$$

$$r^2 = 25$$
$$r = 5$$

$$x^2 + 4x + y^2 = 20$$

$$(x^2 + y^2) + 4x = 20$$

$$r^2 + 4r\cos\theta = 20$$

The long way:

$$(r\cos\theta)^2 + 4\,r\cos\theta + (r\sin\theta)^2 = 20$$

$$r^2\cos^2\theta + 4r\cos\theta + r^2\sin^2\theta = 20$$

$$r^2(\cos^2\theta + \sin^2\theta) + 4r\cos\theta = 20$$
$$r^2 + 4r\cos\theta = 20$$

Parabola:

$$y = 4x^2$$

$$r\sin\theta = 4(r\cos\theta)^2$$

$$r\sin\theta = 4r^2\cos^2\theta$$

$$\sin\theta = 4r\cos^2\theta$$
$$\frac{\frac{1}{4}\sin\theta}{\cos^2\theta} = r$$

$$\frac{\frac{1}{4}\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta} = \frac{1}{4}\tan\theta\sec\theta = r$$

From polar to rectangular:

$$r = 3$$
$$r^2 = 9$$
$$x^2 + y^2 = 9$$

We saw the line through the origin earlier.

$$r = 4\sin\theta$$

Resist the urge to replace theta inside the trig function with inverse tangent.

Multiply by r on both sides

$$r^2 = 4r\sin\theta$$

$$x^2 + y^2 = 4y$$

Example.

$$r^2 = \sin 2\theta$$

$$r^2 = 2\sin\theta\cos\theta$$

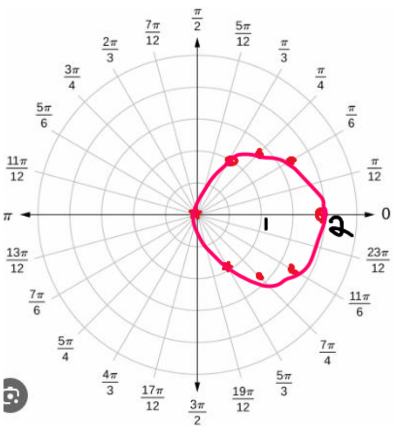
$$r^4 = 2\,r\sin\theta\;\; r\cos\theta$$

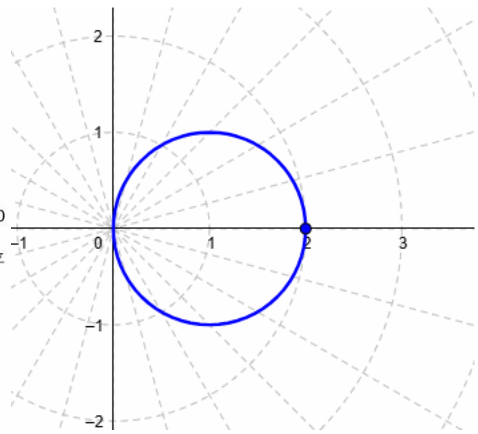
$$(r^2)^2 = (x^2+y^2)^2 = 2xy$$

$$(x^2+y^2)^2 = 2xy$$

Keep in mind, you must simply trig functions as much as possible, and polar functions only contain r and theta, and rectangular functions only contain x and y.

Plotting polar functions.



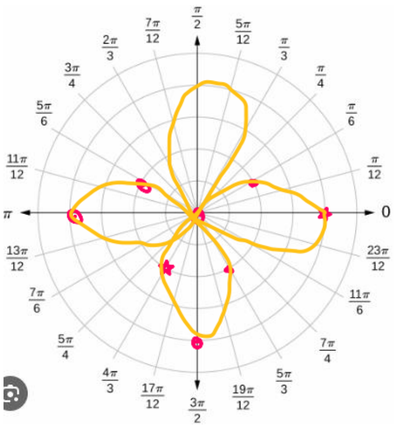


Graph the polar curve $r = 2\cos\theta$

θ	r
0	2
$\frac{\pi}{6}$	$\sqrt{3} \approx 1.7$
$\frac{\pi}{4}$	$\sqrt{2} \approx 1.4$
$\frac{\pi}{3}$	1
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	-1
$\frac{3\pi}{4}$	$-\sqrt{2}$
$\frac{5\pi}{6}$	$-\sqrt{3}$
π	-2

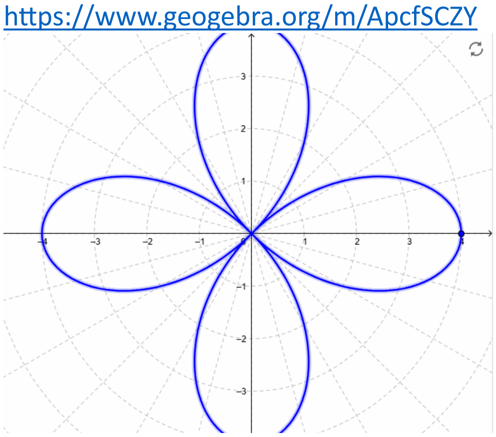
Use symmetry to help reduce the number of points you need to plot:
Cosine is symmetrical across the x-axis, and sine is symmetrical across the y-axis.

Plot the polar curve $r = 4\cos 2\theta$



θ	$r = 4\cos 2\theta$
0	4
$\frac{\pi}{6}$	2
$\frac{\pi}{4}$	0
$\frac{\pi}{3}$	-2
$\frac{\pi}{2}$	-4
$\frac{2\pi}{3}$	-2
$\frac{3\pi}{4}$	0
$\frac{5\pi}{6}$	2
π	4

https://www.geogebra.org/m/ApcfSCZY



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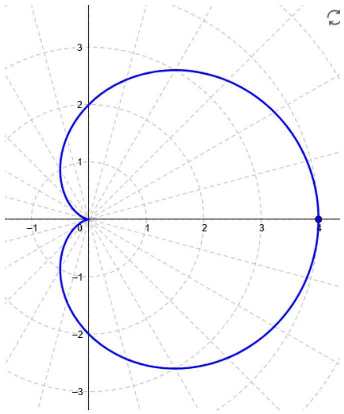

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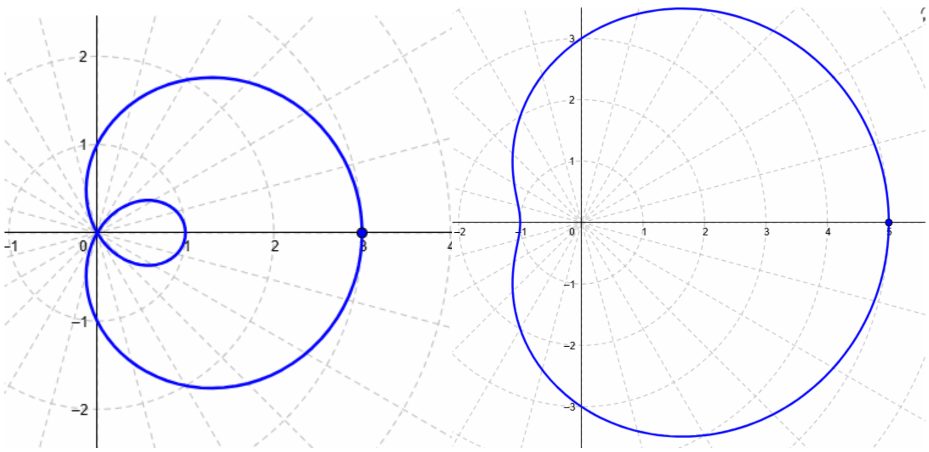
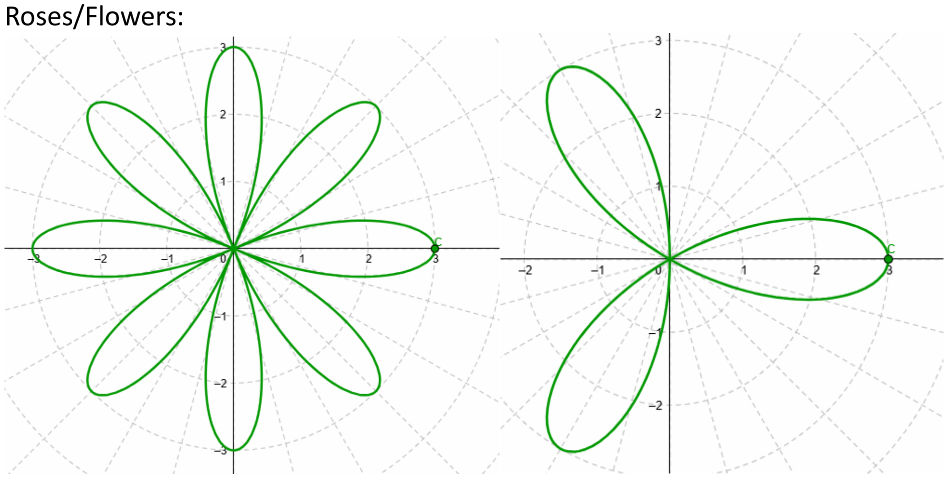


Roses/Flowers:



Next time we'll do complex numbers in polar form.