5/6/2025

Binomial Theorem
Review for Final

Factorial:

$$n! = n(n-1)(n-2)\dots.(2)(1)$$

$$7! = 7(6)(5)(4)(3)(2)(1) = 5040$$

Combinations:
We can't repeat values in a list, and the order doesn't matter.
Permutations:
We can't repeat values in a list, but the order does matter.

Electing officer for a club—two people can't hold the same office, but you can't hold multiple offices.
Choosing a committee membership—everyone on the committee is the same rank, so the order you are selected in doesn't matter

Permutation: 123, 132, 213, 231, 312, 321
Combination: 123

Formulas for calculating the number of permutations or the number of combination, depend on factorial.

$$P(n,r) = P_r^n = nPr = \frac{n!}{(n-r)!}$$

n the number of items in the set we are selecting from
r is the number of items being selected (pick)

$$C(n,r) = C_r^n = nCr = \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

MATH→PRB→
2: nPr
3: nCr
4: !

Find the value of $10P5$

$$10P5 = \frac{10!}{(10-5)!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1} =$$

$$10 \times 9 \times 8 \times 7 \times 6 = 30{,}240$$

10
MATH→PRB→2
5

Screen will look like 10 nPr 5
ENTER
30240

Find the value of $10C5 = \binom{10}{5}$

$$\binom{10}{5} = \frac{10!}{(10-5)!\,5!} = \frac{10!}{5!\,5!} = \frac{10\times 9\times 8\times 7\times 6\times 5\times 4\times 3\times 2\times 1}{(5\times 4\times 3\times 2\times 1)(5\times 4\times 3\times 2\times 1)} =$$

$$\frac{10\times 9\times 8\times 7\times 6}{5\times 4\times 3\times 2\times 1} = \frac{\overset{2}{\cancel{10}}\times 9\times \overset{2}{\cancel{8}}\times 7}{\cancel{5}\times \cancel{4}} = 2\times 9\times 2\times 7 = 252$$

10
MATH→PRB→3
5
Screen will look like 10 nCr 5
ENTER
252

Binomial Theorem

$$(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i}$$

Example

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$\binom{2}{0} a^0 b^{2-0} + \binom{2}{1} a^1 b^{2-1} + \binom{2}{2} a^2 b^{2-2} =$$

$$\binom{2}{0}(1)b^2 + \binom{2}{1} ab + \binom{2}{2} a^2(1) =$$

$$(1)b^2 + 2ab + 1a^2 = a^2 + 2ab + b^2$$

Example:

$$(a+b)^3 = \binom{3}{0} a^3 + \binom{3}{1} a^2 b + \binom{3}{2} ab^2 + \binom{3}{3} b^3 =$$

$$a^3 + 3a^2b + 3ab^2 + b^3$$

Example:

$$(a+b)^7 =$$

$$\binom{7}{0} a^7 + \binom{7}{1} a^6 b^1 + \binom{7}{2} a^5 b^2 + \binom{7}{3} a^4 b^3 + \binom{7}{4} a^3 b^4 + \binom{7}{5} a^2 b^5 + \binom{7}{6} ab^6 + \binom{7}{7} b^7 =$$

$$a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$$

Pascal's triangle

1 ← row 0 $\quad n=0$

1 1 ← $n=1$

1 2 1 ← $n=2$ $\quad \binom{2}{0}=1 \quad \binom{2}{1}=2 \quad \binom{2}{2}=1$

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1 $\quad n=7$

In $(a+b)^{15}$, what is the coefficient of a^7b^8

$$\binom{n}{i}a^ib^{n-i} = \binom{15}{7}a^7b^8 = 6435a^7b^8$$

Example.

$$(2x-3)^4 =$$

$$a = 2x, b = -3, n = 4$$

$$\binom{4}{0}a^4 + \binom{4}{1}a^3b + \binom{4}{2}a^2b^2 + \binom{4}{3}ab^3 + \binom{4}{4}b^4 =$$

$$\binom{4}{0}(2x)^4 + \binom{4}{1}(2x)^3(-3) + \binom{4}{2}(2x)^2(-3)^2 + \binom{4}{3}(2x)(-3)^3 + \binom{4}{4}(-3)^4 =$$

$$(1)(2x)^4 + (4)(2x)^3(-3) + (6)(2x)^2(-3)^2 + (4)(2x)(-3)^3 + (1)(-3)^4 =$$

$$16x^4 - 96x^3 + 216x^2 - 216x + 81$$

The End!

The Final Exam is Thursday!

After you take the final, send me a list of Part 1 problems you would like to have checked for partial credit. Use the numbers from the paper exam, I just a need a list. You don't need to justify anything. Exam #1, and the problem #s

Exam 1, #2, 4a, 6c
Exam 2, #3ai, 5b, 7
Etc.