MTH 162, Quiz #11, Spring 2025 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Answer all parts of each question.

1. Find the product of $z_1 = \cos 70° + i \sin 70°, z_2 = \cos 80° + i \sin 80°$

$$z_1 z_2 = \cos(150°) + i \sin(150°)$$
$$= -\frac{\sqrt{3}}{2} + \frac{1}{2}i$$

2. Divide $\frac{z_1}{z_2}$ *if* $z_1 = \cos 70° + i \sin 70°, z_2 = \cos 80° + i \sin 80°$.

$$\frac{z_1}{z_2} = \cos(70-80°) + i \sin(70-80°)$$
$$= \cos(10°) + i \sin(-10)°$$
$$= \cos 10° - i \sin(10°)$$
$$\approx 0.9848 - 0.1736481i$$

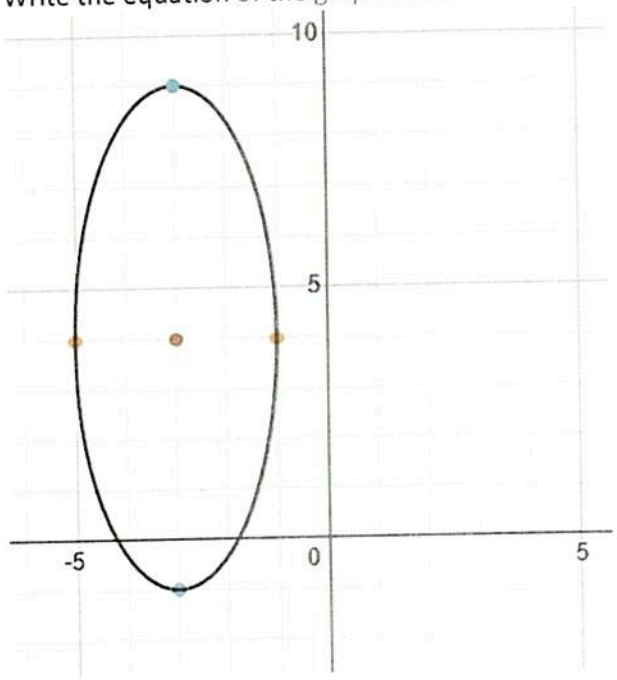
3. Find the complex cube roots of $-1 + i$.

$$z = \sqrt{2}\, e^{3\pi/4 i}$$
$$z_1^{1/3} = \sqrt[6]{2}\, e^{\pi/4 i}$$
$$z_2^{1/3} = \sqrt[6]{2}\, e^{11\pi/12 i}$$
$$z_3^{1/3} = \sqrt[6]{2}\, e^{19\pi/12 i}$$

4. Find $(1 - i)^5$ using DeMoivre's Theorem.

$$z = \sqrt{2}\, e^{7\pi/4 i}$$
$$z^5 = 4\sqrt{2}\, e^{35\pi/4 i}$$
$$= 4\sqrt{2}\, e^{3\pi/4 i}$$
$$= 4\sqrt{2}\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)$$
$$= -4 + 4i$$

5. Write the equation of the graph shown below.

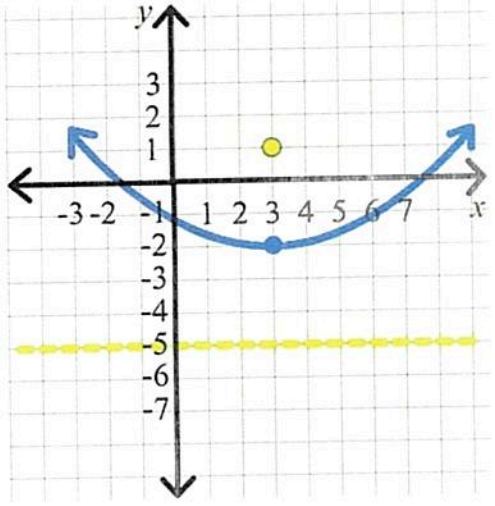


$(h,k) = (-3,4)$

$\uparrow a = 5, \quad b \rightarrow = 2$

$$\frac{(x+3)}{4} + \frac{(y-4)}{25} = 1$$

6. Find the equation of the parabola shown.



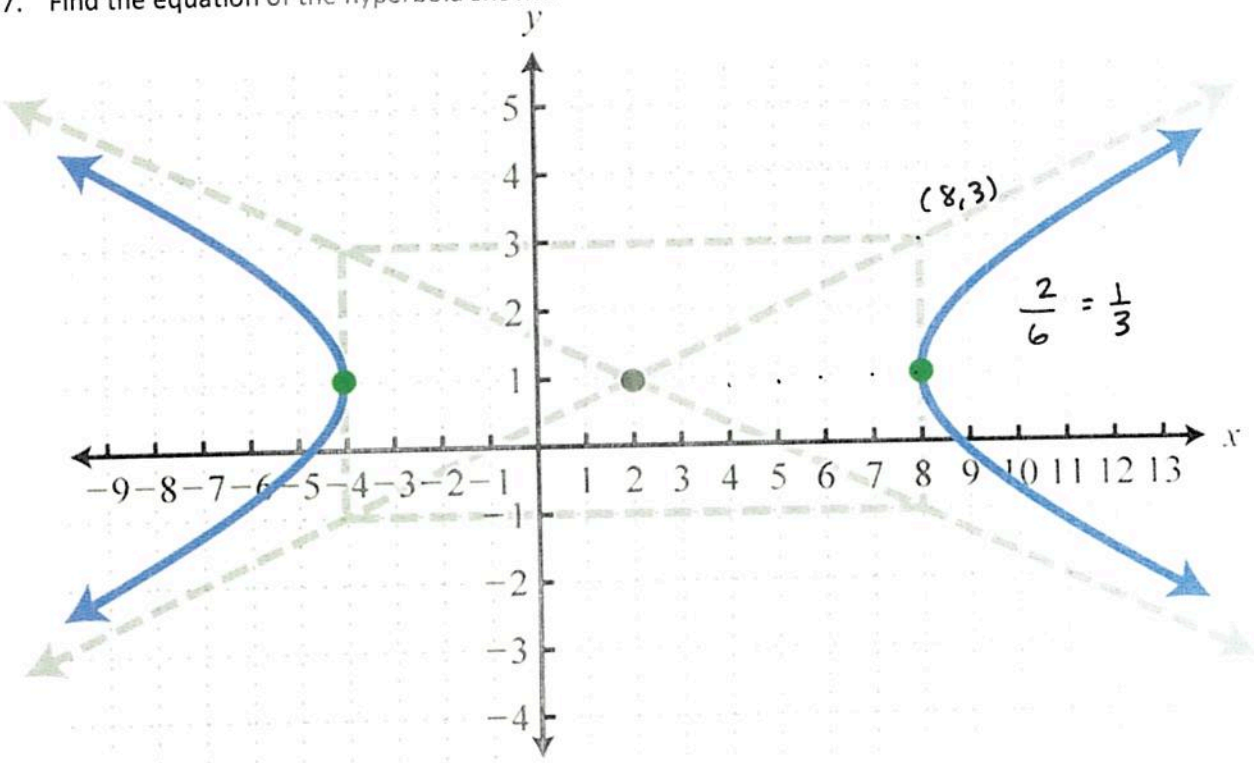
$p = 3$

$(h,k) = (3,-2)$

$12(y+2) = (x-3)^2$

$(h,k) = (2,1)$

7. Find the equation of the hyperbola shown.



$$\frac{(x-2)^2}{36} - \frac{(y-1)^2}{4} = 1$$

8. Graph the equation $r = \frac{2}{3+3\sin\theta}$. Determine the type of conic from the eccentricity.

$\frac{2}{3}\left(\frac{1}{1+\sin\theta}\right)$ parabola