MTH 162, Quiz #13, Spring 2025 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Answer all parts of each question.

1. Find the sum of $\sum_{i=0}^{4} \frac{(-1)^i}{i!}$.

$$\frac{(-1)^0}{0!} + \frac{(-1)^1}{1!} + \frac{(-1)^2}{2!} + \frac{(-1)^3}{3!} + \frac{(-1)^4}{4!}$$

$$1 + (-1) + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} = \frac{3}{8}$$

2. Use mathematical induction to prove that $\sum_{i=1}^{n}(4i-1) = n(2n+1)$.

base case

$i=1$, $n=1$ $\quad 4(1)-1 = 3 \qquad (1)(2(1)+1) = 3$ ✓

Suppose true for n, show for $n+1$

$$\sum_{i=1}^{n+1}(4i-1) = \sum_{i=1}^{n}(4i-1) + 4(n+1)-1 = n(2n+1) + 4(n+1) - 1$$

$$2n^2 + n + 4n + 4 - 1 =$$

$$2n^2 + 5n + 3 = (2n+3)(n+1)$$

Show = $(n+1)[2(n+1)+1]$

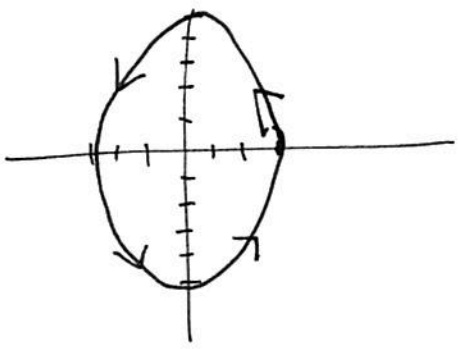
$$= (n+1)(2(n+1)+1) \quad ✓$$

3. Use the binomial theorem to expand $(x^2 - y)^4$.

$$(x^2)^4 + 4(x^2)^3(-y) + 6(x^2)^2(-y)^2 + 4(x^2)(-y)^3 + (-y)^4$$

$$x^8 - 4x^6y + 6x^4y^2 - 4x^2y^3 + y^4$$

4. Draw the graph of $x = 3\cos t$, $y = 5\sin t$. Note the orientation on the graph.



5. Convert the function $y = 3x - 4$ into a parametric equation in two different ways. Describe how the two results differ.

answers will vary

$X = t$
$Y = 3t - 4$

Simplest way
covers the whole graph
crosses through at a steady rate

$X = t^2$
$Y = 3t^2 - 4$

only does positive part of graph
passes through at an increasing rate

$X = \sin t$
$Y = 3\sin t - 4$

only does a section of the graph
where $x \in [-1, 1]$

etc.