

KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Answer all parts of each question.

1. Prove the identities.

a. $\cos\left(\frac{3\pi}{2} - x\right) = -\sin x$

$$\begin{aligned}\cos(\alpha - \beta) &= \cos \alpha \cos \beta + \sin \alpha \sin \beta \\ &= \cos\left(\frac{3\pi}{2}\right) \cos x + \sin\left(\frac{3\pi}{2}\right) \sin x \\ &= (-1) \sin x = -\sin x \checkmark\end{aligned}$$

b. $\frac{\csc x - \sec x}{\csc x + \sec x} = \frac{\cot x - 1}{\cot x + 1}$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\frac{\frac{1}{\sin x} - \frac{1}{\cos x}}{\frac{1}{\sin x} + \frac{1}{\cos x}} \cdot \frac{\sin x \cos x}{\sin x \cos x} = \frac{\frac{\sin x \cos x}{\sin x} - \frac{\sin x \cos x}{\cos x}}{\frac{\sin x \cos x}{\sin x} + \frac{\sin x \cos x}{\cos x}} = \frac{\cos x - \sin x}{\cos x + \sin x}$$

$$* \frac{1}{\sin x} = \frac{\cot x - 1}{\cot x + 1} \checkmark$$

c. $\frac{\sin \theta - \cos \theta}{\sin \theta} + \frac{\cos \theta - \sin \theta}{\cos \theta} = 2 - \sec \theta \csc \theta$

$$\frac{\sin \theta}{\sin \theta} - \frac{\cos \theta}{\sin \theta} + \frac{\cos \theta}{\cos \theta} - \frac{\sin \theta}{\cos \theta} = 2 - \cot \theta - \tan \theta$$

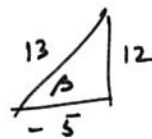
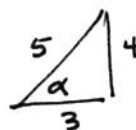
$$\begin{aligned}\cot \theta + \tan \theta &= \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} = \frac{\cos^2 \theta}{\sin \theta \cos \theta} + \frac{\sin^2 \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta} \\ &= \sec \theta \csc \theta\end{aligned}$$

$$\Rightarrow 2 - \cot \theta - \tan \theta = 2 - \sec \theta \csc \theta \checkmark$$

2. Simplify and/or evaluate $\cos\left[\tan^{-1}\left(\frac{4}{3}\right) + \cos^{-1}\left(-\frac{5}{13}\right)\right]$. $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

$$\begin{aligned}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right) \cdot \cos\left(\cos^{-1}\left(-\frac{5}{13}\right)\right) - \sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right) \cdot \sin\left(\cos^{-1}\left(-\frac{5}{13}\right)\right) \\ \left(\frac{3}{5}\right) \left(-\frac{5}{13}\right) - \left(\frac{4}{5}\right) \left(\frac{12}{13}\right)\end{aligned}$$

$$-\frac{3}{13} - \frac{48}{65} = -\frac{63}{65}$$



in Q2
sin is +