MTH 162, Quiz #7, Spring 2025 Name____

Instructions: Show all work. Use exact answers unless specifically asked to round. Answer all parts of each question.

1. Rewrite $\sin 8x \sin 4x$ as a sum or difference.

2. Prove the identity.
   a. $1 - \tan^2 x = \frac{\cos 2x}{\cos^2 x}$

   b. $\frac{\sin x - \sin y}{\cos x - \cos y} = -\cot\left(\frac{x+y}{2}\right)$

3. Simplify and/or evaluate $\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$

4. Rewrite the expression $\cos^6 x \sin^2 x$ as a sum of only linear terms (hint: use the power-reducing identity; you may need it several times).

Some useful formulas:

$$\sin(a+b)=\sin a\cos b+\sin b\cos a$$
$$\sin(a-b)=\sin a\cos b-\sin b\cos a$$
$$\cos(a+b)=\cos a\cos b-\sin a\sin b$$
$$\cos(a-b)=\cos a\cos b+\sin a\sin b$$
$$\tan(a+b)=\frac{\tan a+\tan b}{1-\tan a\tan b}$$
$$\tan(a-b)=\frac{\tan a-\tan b}{1+\tan a\tan b}$$

$$\begin{aligned}\sin 2\alpha &= 2\sin\alpha\cos\alpha\\ \cos 2\alpha &= \cos^2\alpha-\sin^2\alpha\\ &= 2\cos^2\alpha-1\\ &= 1-2\sin^2\alpha\\ \tan 2\alpha &= \frac{2\tan\alpha}{1-\tan^2\alpha}\end{aligned}$$

$$\sin(\frac{a}{2})=\pm\sqrt{\frac{(1-\cos a)}{2}}$$
$$\cos(\frac{a}{2})=\pm\sqrt{\frac{(1+\cos a)}{2}}$$
$$\tan(\frac{a}{2})=\frac{1-\cos a}{\sin a}=\frac{\sin a}{1+\cos a}$$

$$\sin^2\theta=\frac{1-\cos 2\theta}{2}$$
$$\cos^2\theta=\frac{1+\cos 2\theta}{2}$$
$$\tan^2\theta=\frac{1-\cos 2\theta}{1+\cos 2\theta}$$

$$\cos(a)\cos(b) = \frac{1}{2}\left(\cos(a+b)+\cos(a-b)\right)$$
$$\sin(a)\sin(b) = \frac{1}{2}\left(\cos(a-b)-\cos(a+b)\right)$$
$$\sin(a)\cos(b) = \frac{1}{2}\left(\sin(a+b)+\sin(a-b)\right)$$
$$\cos(a)\sin(b) = \frac{1}{2}\left(\sin(a+b)-\sin(a-b)\right)$$

$$\sin(A)+\sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\sin(A)-\sin(B) = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$
$$\cos(A)+\cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos(A)-\cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$