MTH 162, Quiz #7, Spring 2025 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Answer all parts of each question.

1. Rewrite $\sin 8x \sin 4x$ as a sum or difference.

$\alpha = 8x,\ \beta = 4x$

$$\sin\alpha \sin\beta = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right)$$
$$= \frac{1}{2}\left[\cos 4x - \cos 12x\right]$$

2. Prove the identity.

a. $1-\tan^2 x = \frac{\cos 2x}{\cos^2 x} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} - \frac{\sin^2 x}{\sin^2 x} = 1 - \tan^2 x$ ✓

b. $\frac{\sin x - \sin y}{\cos x - \cos y} = -\cot\left(\frac{x+y}{2}\right) = \frac{\cancel{2}\cos\left(\frac{x+y}{2}\right)\cancel{\sin\left(\frac{x-y}{2}\right)}}{-\cancel{2}\sin\left(\frac{x+y}{2}\right)\cancel{\sin\left(\frac{x-y}{2}\right)}} =$

$-\cot\left(\frac{x+y}{2}\right)$

3. Simplify and/or evaluate $\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}} = \tan\left(2\cdot\frac{\pi}{8}\right) = \tan\frac{\pi}{4} = 1$

4. Rewrite the expression $\cos^6 x \sin^2 x$ as a sum of only linear terms (hint: use the power-reducing identity; you may need it several times).

$(\cos^2 x)^3 \sin^2 x = \left(\frac{1}{2}(1+\cos 2x)\right)^3 (1-\cos 2x)\cdot\frac{1}{2} = \frac{1}{16}(1+\cos 2x)^2(1-\cos^2 2x)$

$= \frac{1}{16}(1+2\cos 2x+\cos^2 2x)(1-\cos^2 2x) = \frac{1}{16}\left(1-\cancel{\cos^2 2x}+2\cos 2x-2\cos^3 2x+\cancel{\cos^2 2x}-\cos^4 2x\right)$

$= \frac{1}{16}\left(1-2\cos 2x-\cancel{2}\cos 2x\cdot\left(\cancel{\tfrac{1}{2}}(1+\cos 4x)\right)-\left[\tfrac{1}{2}(1+\cos 4x)\right]^2\right) = \frac{1}{16}\left(1-2\cos 2x-\cos 2x-\right.$

$\left.\cos 2x\cos 4x-\frac{1}{4}(1+2\cos 4x+\cos^2 4x)\right) = \frac{1}{16}\left(1-3\cos 2x-\frac{1}{2}(\cos 6x+\cos 2x)-\frac{1}{4}-\frac{1}{2}\cos 4x\right.$

$\left.-\frac{1}{4}\cos^2 4x\right) = \frac{1}{16}\left(\frac{3}{4}-\frac{7}{2}\cos 2x-\frac{1}{2}\cos 6x-\frac{1}{4}\cos 4x-\frac{1}{8}(1+\cos 8x)\right) =$