

Name KEY
Math 254, Quiz #10, Summer 2012

Instructions: Show all work. Use exact answers unless asked to round.

1. Find the area of the surface of the function $f(x, y) = xy$ over the region $R = \{(x, y): x^2 + y^2 \leq 16\}$. You may find it convenient to convert the integral to polar coordinates to complete the integration.

$$f_x = y$$
$$f_y = x$$

$$S = \iint_R \sqrt{1 + y^2 + x^2} \, dA = \int_0^{2\pi} \int_0^4 \sqrt{1 + r^2} \, r \, dr \, d\theta$$

$$u = 1 + r^2$$
$$du = 2r \, dr$$

$$\int_0^{2\pi} \frac{1}{2} (1 + r^2)^{3/2} \cdot \frac{2}{3} \, d\theta =$$

$$\frac{1}{3} (1 + r^2)^{3/2} \Big|_0^4 \cdot 2\pi$$

$$\boxed{\frac{2\pi}{3} [17^{3/2} - 1]}$$