Name KEY

Math 254, Quiz #3, Summer 2012

Instructions: Show all work. Use exact answers unless the problem specifically asks you to approximate or begins with decimal values.

1. Find the unit tangent vector for the vector-valued function $\vec{r}(t) = \cos(t)\,\hat{\imath} + 2\sin(t)\,\hat{\jmath} + \hat{k}$. What are the two possible values for the principle unit normal vector (without doing derivatives)? What is the equation for finding the binormal? What is the binormal vector? (You can find this without doing any calculations.)

$r'(t) = -\sin t\,\hat{\imath} + 2\cos t\,\hat{\jmath}$

$\|r'(t)\| = \sqrt{\sin^2 t + 4\cos^2 t} = \sqrt{1+3\cos^2 t}$

$$T(t) = \frac{-\sin t\,\hat{\imath} + 2\cos t\,\hat{\jmath}}{\sqrt{1+3\cos^2 t}}$$

$N(t)$ either $\dfrac{-2\cos t\,\hat{\imath} - \sin t\,\hat{\jmath}}{\sqrt{1+3\cos^2 t}}$ or $\dfrac{2\cos t\,\hat{\imath} + \sin t\,\hat{\jmath}}{\sqrt{1+3\cos^2 t}}$

$B(t) = N(t) \times T(t)$ or $T(t) \times N(t)$

$= \hat{k}$ (or possibly $-\hat{k}$) since T & N both in $\hat{\imath}/\hat{\jmath}$ only

2. What is the equation for the curvature of the function $\vec{r}(t) = t^2\hat{\imath} + \ln(t)\,\hat{\jmath} + t^3\hat{k}$. What is the radius of curvature at the value t=1.

$r'(t) = 2t\,\hat{\imath} + \frac{1}{t}\,\hat{\jmath} + 3t^2\,\hat{k}$

$r''(t) = 2\,\hat{\imath} - \frac{1}{t^2}\,\hat{\jmath} + 6t\,\hat{k}$

$$r'(t) \times r''(t) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & k \\ 2t & \frac{1}{t} & 3t^2 \\ 2 & -\frac{1}{t^2} & 6t \end{vmatrix} = (6+3)\hat{\imath} - (12t^2 - 6t^2)\hat{\jmath} + \left(-\frac{2}{t} - \frac{2}{t}\right)\hat{k}$$

$$9\,\hat{\imath} - 6t^2\,\hat{\jmath} - \frac{4}{t}\,\hat{k}$$

$\|r'(t) \times r''(t)\| = \sqrt{81 + 36t^4 + \frac{16}{t^2}}$

$\|r'(t)\| = \sqrt{4t^2 + \frac{1}{t^2} + 9t^4}$

$$K = \frac{\sqrt{81 + 36t^4 + \frac{16}{t^2}}}{\left[4t^2 + \frac{1}{t^2} + 9t^4\right]^{3/2}}$$

$K(1) = \dfrac{\sqrt{81+36+16}}{(\sqrt{4+1+9})^3} = \dfrac{\sqrt{133}}{\sqrt{14}^3} \approx .220$ $\quad R(1) \approx \dfrac{\sqrt{14}^3}{\sqrt{133}} \approx 4.54$