

Name _____

KEY

Math 254, Quiz #6, Summer 2012

Instructions: Show all work. Use exact answers unless asked to round.

1. Find the tangent plane to the graph $\ln(xy) + x^2 - 4y^3 + 2xz = 3$ at the point $(1,1,3)$. Then write the equation of the normal line to the graph at the same point in symmetric form.

$$\nabla F = \left\langle \frac{1}{x} + 2x + 2z, \frac{1}{y} - 12y^2, 2x \right\rangle$$

$$\begin{array}{ccc} 1 + 2 + 6 & 1 - 12 & 6 \\ \langle 9 & -11 & 6 \rangle \end{array}$$

Plane: $9(x-1) + (-11)(y-1) + 6(z-3) = 0$

line: $\frac{x-1}{9} = \frac{y-1}{-11} = \frac{z-3}{6}$

2. Find and characterize any critical points as maxima, minima or saddle points (if it can be determined) to the graph $f(x,y) = x^3 - 3xy - 2y + y^2 - 16$.

$$f_x = 3x^2 - 3y = 0 \Rightarrow y = x^2$$

$$f_y = -3x - 2 + 2y = 0 \Rightarrow -3x - 2 + 2x^2 = 0$$

$$2x^2 - 3x - 2 = 0$$

$$(2x+1)(x-2) = 0$$

$$x = -\frac{1}{2} \quad x = 2$$

$$\downarrow \quad \downarrow$$

$$y = \frac{1}{4} \quad y = 4$$

$$f_{xx} = 6x$$

$$f_{yy} = 2$$

$$f_{xy} = -3$$

$$D(-\frac{1}{2}, \frac{1}{4}) = (-3)(2) - (-3)^2 = -6 - 9 = -15$$

$$D(2, 4) = (12)(2) - (-3)^2 = 24 - 9 = 15$$

$$(-\frac{1}{2}, \frac{1}{4}) \quad (2, 4)$$

Saddle min