Name KEY

Math 254, Quiz #8, Summer 2012

Instructions: Show all work. Use exact answers unless asked to round.

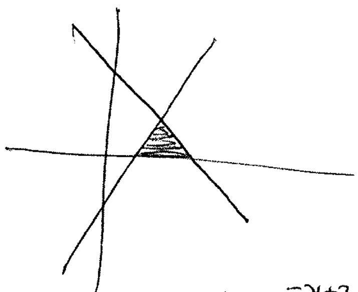
1. Find the volume of the solid formed by integrating the function $f(x, y) = x + \sin(xy)$ over the region bounded by the graphs y=0, x+y=2 and 2x-y=1.

$y = -x+2 \quad 2x-1 = y$

$x = -y+2$

$\frac{2x}{2} = \frac{y+1}{2}$

$x = \frac{y+1}{2}$



$$\int_0^1 \int_{\frac{1}{2}(y+1)}^{-y+2} x + \sin(xy)\, dx\, dy$$

$$\boxed{\approx 1.1396}$$

$$\int_0^1 \frac{x^2}{2} + \frac{-1}{y}\cos(xy) \Big|_{\frac{1}{2}(y+1)}^{-y+2} = \int_0^1 \frac{(-y+2)^2}{2} - \frac{1}{y}\left(\cos(y(-y+2))\right) - \frac{1}{8}(y+1)^2 + \frac{1}{y}\cos\left(\tfrac{1}{2}(y)(y+1)\right) dy$$

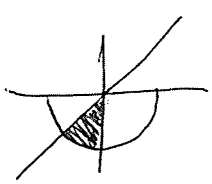
$-y^2+2y \qquad y^2+y$

2. Convert the integral to polar coordinates to integrate. Sketch the region for each.

a. $\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sqrt{x^2+y^2}\, dy\, dx$

$$\int_{-\pi/2}^{\pi/2} \int_0^2 r^2\, dr\, d\theta = \int_{-\pi/2}^{\pi/2} \frac{r^3}{3}\Big|_0^2 d\theta = \int_{-\pi/2}^{\pi/2} \frac{8}{3}\, d\theta = \boxed{\frac{8}{3}\pi}$$

b. $\int_{-3\sqrt{2}/2}^{0} \int_{-\sqrt{9-x^2}}^{x} dy\, dx$



$$\int_{-3\pi/4}^{-\pi/2} \int_0^3 r\, dr\, d\theta = \int_{-3\pi/4}^{-\pi/2} \frac{r^2}{2}\Big|_0^3 d\theta =$$

$$\int_{-3\pi/4}^{-\pi/2} \frac{9}{2}\, d\theta = \frac{9}{2} \cdot \frac{\pi}{4} = \boxed{\frac{9\pi}{8}}$$