Name KEY

Math 255, Quiz #3, Summer 2012

Instructions: Show all work. Use exact answers unless the problem specifically asks you to approximate or begins with decimal values.

1. Determine whether the equations are linear, separable, exact, homogeneous or Bernoulli.

   a. $(1+t^2)y' + 4ty = (1+t^2)^{-2}$

   $y' + \frac{4t}{1+t^2} y = (1+t^2)^{-3}$

   linear

   b. $y' + 3x^2y = x^2y^3$

   Bernoulli $n=3$

   (can be separated, too $\frac{dy}{y^3-3y} = x^2\,dx$ – needs partial fractions though)

   c. $y' = \frac{x^2+y^2}{2xy}$

   homogeneous

   $\frac{t^2x^2 + t^2y^2}{2txty} = \frac{t^2(x^2+y^2)}{t^2(2xy)}$ ✓

   d. $(e^x \sin y + 3y)dx + (3x + e^x \cos y)dy = 0$

   $M_y = e^x \cos y + 3 \qquad N_x = 3 + e^x \cos y$

   exact

   $M_y = N_x$

   e. $\sin 2x\,dx + \cos 3y\,dy = 0$

   separable

   $\cos 3y\,dy = -\sin 2x\,dx$

2. Solve the linear differential equation $y' - 2y = e^{3x}$ for the general solution. If the initial condition is y(0)=2, find the value of any constants.

$\mu = e^{\int -2\,dx} = e^{-2x}$

$e^{-2x}y' - 2e^{-2x}y = e^x$

$(e^{-2x}y)' = e^x$

$e^{-2x}y = \int e^x\,dx = e^x + C$

$y = e^{3x} + Ce^{+2x}$

agree

$\mu = e^{-2x}$

$y = e^{2x}\int e^{-2x}e^{3x}\,dx$

$= e^{2x}\int e^x\,dx$

$= e^{2x}[e^x + C]$

$= e^{3x} + Ce^{2x}$

$2 = e^0 + Ce^0$

$2 = 1 + C$

$C = 1$

$\boxed{y = e^{3x} + e^{2x}}$