Name KEY

Math 255, Quiz #6, Summer 2012

Instructions: Show all work. Use exact answers unless asked to round.

1. The solutions $y_1 = e^{3x}\sin(2x), y_2 = e^{3x}\cos(2x)$ are solutions to the differential equation $y'' - 6y' + 13y = 0$. Verify, by using the Wronskian, that the solutions form a fundamental set.

$$W = \begin{vmatrix} e^{3x}\sin 2x & e^{3x}\cos 2x \\ 3e^{3x}\sin 2x + 2e^{3x}\cos 2x & 3e^{3x}\cos 2x - 2e^{3x}\sin 2x \end{vmatrix} =$$

$$\cancel{3e^{6x}\cos 2x \sin 2x} - 2e^{6x}\sin^2 2x - \cancel{3e^{6x}\sin 2x \cos 2x} - 2e^{6x}\cos^2 2x$$

$$-2e^{6x}\underbrace{(\sin^2 2x + \cos^2 2x)}_{=1} = -2e^{2x} \neq 0 \checkmark$$

2. Write the characteristic equation for the differential equation $y'' - 6y' + 13y = 0$, and verify that the solutions given above ARE the solutions to that equation. (The solutions to the characteristic equation are complex. Use the quadratic formula to find them, and use appropriate conversion formulas to obtain the solutions above.)

$$r^2 - 6r + 13 = 0$$

$$r = \frac{6 \pm \sqrt{36 - 4(13)}}{2} = \frac{6 \pm \sqrt{36 - 52}}{2} = \frac{6 \pm \sqrt{-16}}{2} =$$

$$\frac{6 \pm 4i}{2} = 3 \pm 2i$$

$$\therefore\ Y_1 = e^{3x}\cos 2x;\ Y_2 = e^{3x}\sin 2x$$