Math 1020, Exam #3, Summer 2013 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Decimal answers in other cases will not receive full credit. If you are using your calculator graphing features to obtain a result, sketch the graph (label the window size and any key points like intercepts, etc.).

1. Solve the following inequalities. Write the answer to each in the **three** solution formats: i) in set notation, ii) graphed on a number line, iii) written in interval notation. (10 points each)

a. $2(3x-8)+7 \geq 4x+10$

$6x-16+7 \geq 4x+10$

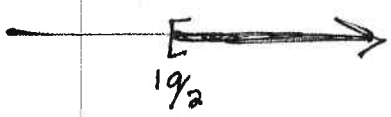
$6x-9 \geq 4x+10$

$-4x+9 \quad -4x+9$

$\frac{2x}{2} \geq \frac{19}{2}$

$x \geq 19/2$

i) $\{x \mid x \geq 19/2\}$

ii) 

iii) $[19/2, \infty)$

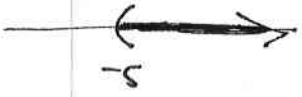
b. $-5x+3<28$

$-3 \quad -3$

$\frac{-5x}{-5} < \frac{25}{-5}$

$x > -5$

i) $\{x \mid x > -5\}$

ii) 

iii) $(-5, \infty)$

c. $\left(\frac{1}{4}(2x-7) \leq \frac{2}{3}\left(3x+\frac{5}{4}\right)-\frac{3}{2}x\right)12$

$3(2x-7) \leq 8\left(3x+\frac{5}{4}\right)-18x$

$6x-21 \leq 24x+10-18x$

$6x-21 \leq 6x+10$

$-6x \quad -6x$

$-21 \leq 10$ true

i) $\{x \mid x \text{ is real}\}$

ii) 

iii) $(-\infty, \infty)$

d. $9(n+2) - 4n < 5(n+1)$

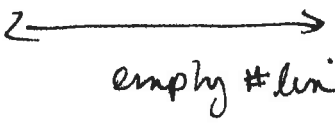
$9n + 18 - 4n < 5n + 5$

$5n + 18 < 5n + 5$

$-5n \quad -5n$

$18 < 5$ false

i) $\{x \mid x$ has no value$\}$
no solution or $\emptyset$

ii) 

iii) $\emptyset$

2. State the values of the points plotted on the graph below. Write each as an ordered pair. (14 points)

A. (6,6)

B. (0,3)

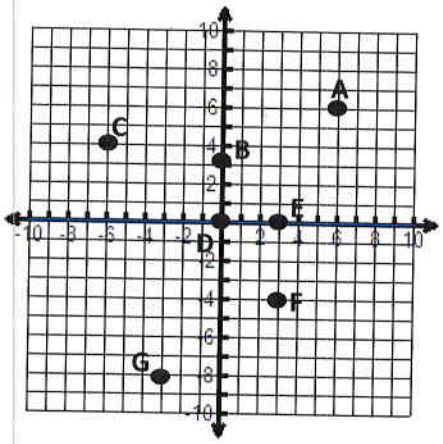
C. (-6,4)

D. (0,0)

E. (3,0)

F. (3,-4)

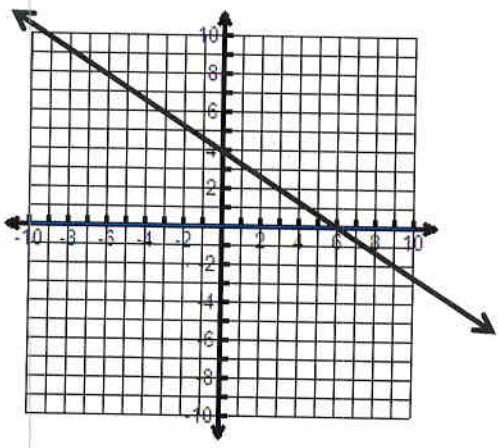
G. (-3,-8)



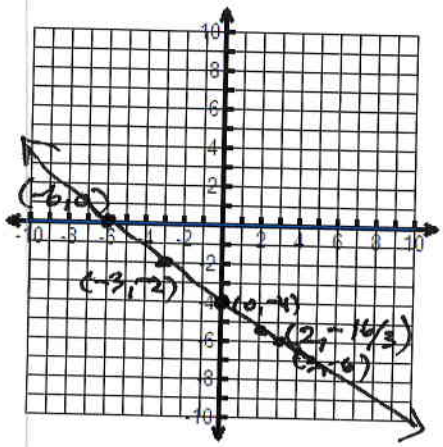
3. For the graph shown to the right, find the locations of the x- and y-intercepts. Be sure to indicate which is which. (6 points)

y-intercept at y = 4 or (0,4)

x-intercept at x = 6 or (6,0)



4. Plot the equation of the line $4x + 6y = -24$ on the graph to the right. Label at least 5 points on the graph, including the x- and y-intercepts. Write the equation of the line in slope-intercept form. (10 points)



$x=0 \quad y=-4 \quad (0,-4)$

$y=0 \quad x=-6 \quad (-6,0)$

$x=2 \quad 8+6y=-24 \quad (2,-16/3)$

$-8 \qquad -8$

$6y=-32$

$y=-32/6\,x-5\frac{1}{3}$

$x=3 \quad 12+6y=-24 \quad (3,-6)$

$-12 \qquad -12$

$6y=-36$

$y=-6$

$x=-3 \quad -12+6y=-24 \quad (-3,-2)$

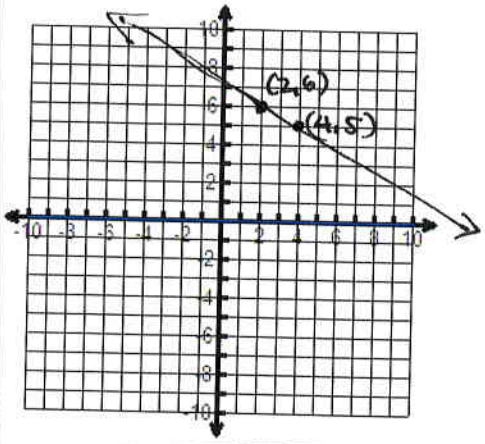
$+12 \qquad +12$

$6y=-12 \quad y=-2$

$\frac{6y}{6}=\frac{-4x}{6}-\frac{24}{6}$

$y=-\frac{2}{3}x-4$

5. Given the points (2,6) and (4,5), draw the line connecting them, and then find the equation of the line that connects them. (10 points)



$m=\frac{6-5}{2-4}=\frac{1}{-2}=-\frac{1}{2}$

$y=mx+b$

$6=-\frac{1}{2}(2)+b \Rightarrow 6=1+b \quad b=7$

$+1 \quad +1$

$y=-\frac{1}{2}x+7$

or $y-y_1=m(x-x_1)$

$y-6=-\frac{1}{2}(x-2) \Rightarrow y-6=-\frac{1}{2}x+1$

$+6 \qquad +6$

$\Rightarrow y=-\frac{1}{2}x+7$

6. Find the slope of the line through the points $\left(\frac{1}{3},\frac{1}{4}\right)$ and $\left(\frac{5}{6},\frac{7}{8}\right)$. Write the equation of the line connecting them in standard form. (10 points)

$y_2-y_1=\frac{1}{4}-\frac{7}{8}=\frac{2}{8}-\frac{7}{8}=-\frac{5}{8}$

$x_2-x_1=\frac{1}{3}-\frac{5}{6}=\frac{2}{6}-\frac{5}{6}=-\frac{3}{6}=-\frac{1}{2}$

$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-5/8}{-1/2}=-\frac{5}{8}\div\left(-\frac{1}{2}\right)=+\frac{5}{8}\times\frac{+2}{1}=\frac{5}{4}$

$y-\frac{1}{4}=\frac{5}{4}\left(x-\frac{1}{3}\right)$

$y-\frac{1}{4}=\frac{5}{4}x-\frac{5}{12}$

$+\frac{1}{4} \qquad +\frac{3}{12}$

$y=\frac{5}{4}x+\frac{2}{12}=\frac{5}{4}x-\frac{1}{6} \quad (\times 12)$

$12y=15x-2 \Rightarrow$

$15x-12y=2$

7. What is the slope of a horizontal line? Explain why in your own words. (7 points)

$m = 0$

because the y-values aren't changing so in $m = \frac{y_2 - y_1}{x_2 - x_1}$,

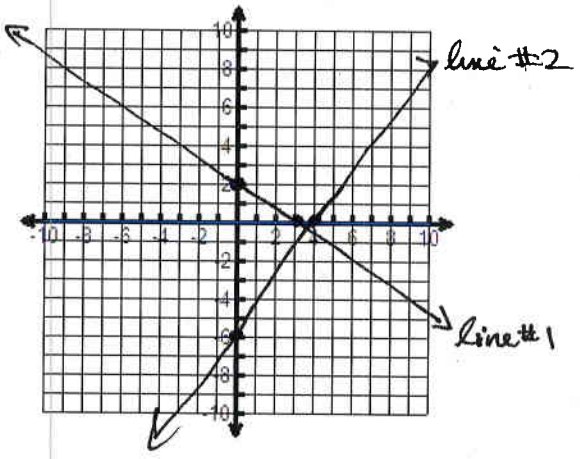
$y_2 - y_1 = 0$ all the time.

8. What is the slope of a vertical line? Give an example of an equation of a vertical line. (7 points)

m is undefined.

$x = 4$

9. Two lines are given: $2x + 3y = 6$ and $3x - 2y = 12$. Graph the two lines on the graph shown. Are they parallel or perpendicular? What features of the equation in slope-intercept form tells you that this would be the case without the graph? (10 points)



$2x + 3y = 6$

$x = 0 \quad y = 2$

$y = 0 \quad x = 3$

$3x - 2y = 12$

$x = 0 \quad y = -6$

$y = 0 \quad x = 4$

they look perdendicular

$\frac{3y}{3} = \frac{-2x + 6}{3} \Rightarrow y = -\frac{2}{3}x + 2$

$\frac{-2y}{-2} = \frac{-3x + 12}{-2} \Rightarrow y = \frac{3}{2}x - 6$

The slopes in this form are negative reciprocals

or $(-2/3)(\frac{3}{2}) = -1$

10. Determine for each pair of equations below if the lines are parallel, perpendicular or neither. (7 points each)

a. $y = \frac{4}{5}x - 1$ and $4x - 5y = 10$ $\Rightarrow$ $\frac{-5y}{-5} = \frac{-4x+10}{-5}$ $\Rightarrow$ $y = \frac{4}{5}x - 2$

the slopes are the same therefore parallel

b. $y = 2x + 11$ and $2x - 3y = 7$ $\Rightarrow$ $\frac{-3y}{-3} = \frac{-2x+7}{-3}$ $\Rightarrow$ $y = \frac{2}{3}x - \frac{7}{3}$

$m=2$ vs. $m = \frac{2}{3}$

neither

c. $y = 3x + 6$ and $x - 3y = 9$ $\Rightarrow$ $\frac{-3y}{-3} = \frac{-x+9}{-3}$ $\Rightarrow$ $y = \frac{1}{3}x - 3$

$m=3$ vs. $m = \frac{1}{3}$

neither

not negative reciprocals, not same

11. These are some problems from the last exam that were most often incorrect. Here's your chance for a second chance at them for extra points.

a. Solve the equation $Q = \frac{1}{3}r(m - p)$, for p. (4 points)

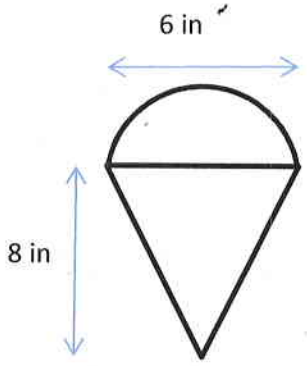
$3Q = rm - rp$

$\frac{3Q - rm}{-r} = \frac{-rp}{-r}$

$\boxed{\frac{rm - 3Q}{r} = p}$

or $m - \frac{3Q}{r} = p$

b. Find the area of the shape. (6 points)



$A_{semicircle} = \frac{\pi r^2}{2} = \pi(3)^2 = \frac{9\pi}{2}$

$A_{triangle} = \frac{1}{2}bh = \frac{1}{2}(6)(8) = 24$

total area $= \frac{9\pi}{2} + 24 \approx 38.14$ in^2