Math 2568, Quiz #11, Summer 2013 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. All answers must be justified with work or written explanation.

1. Find the eigenvalues and eigenvectors of the following matrices.

   a. $\begin{bmatrix} 3 & 2 \\ 3 & 4 \end{bmatrix}$

$$\begin{vmatrix} 3-\lambda & 2 \\ 3 & 4-\lambda \end{vmatrix} = (3-\lambda)(4-\lambda) - 6 = 12 - 7\lambda + \lambda^2 - 6 =$$

$$\lambda^2 - 7\lambda + 6 = 0 \qquad (\lambda-1)(\lambda-6) = 0$$

$$\lambda = 1,\ \lambda = 6$$

$\lambda_1 = 1$ $\quad \begin{vmatrix} 3-1 & 2 \\ 3 & 4-1 \end{vmatrix} = \begin{vmatrix} 2 & 2 \\ 3 & 3 \end{vmatrix}$ $\quad \cancel{2}x_1 = -\cancel{2}x_2$

$x_2 = x_2$

$$\vec{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$\lambda_2 = 6$ $\quad \begin{vmatrix} 3-6 & 2 \\ 3 & 4-6 \end{vmatrix} = \begin{vmatrix} -3 & 2 \\ 3 & -2 \end{vmatrix}$ $\quad \begin{matrix} -3x_1 = -2x_2 \\ x_2 = x_2 \end{matrix} \Rightarrow \begin{matrix} x_1 = \frac{2}{3}x_2 \\ x_2 = x_2 \end{matrix}$

$$\vec{v}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

   b. $\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 0 & -3 & -1 \end{bmatrix}$ $\quad \begin{vmatrix} 1-\lambda & 1 & 1 \\ 2 & -1-\lambda & 1 \\ 0 & -3 & -1-\lambda \end{vmatrix} = (1-\lambda)\begin{vmatrix} -1-\lambda & 1 \\ -3 & -1-\lambda \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ -3 & -1-\lambda \end{vmatrix}$

$$= (1-\lambda)[(-1-\lambda)^2 + 3] - 2[(-1-\lambda) + 3] =$$

$$(1-\lambda)[\lambda^2 + 2\lambda + 4] - 2[-\lambda + 2] =$$

$$\lambda^2 \cancel{+2\lambda} \cancel{+4} - \lambda^3 - 2\lambda^2 \cancel{-4\lambda} \cancel{+2\lambda} \cancel{-4} =$$

$$-\lambda^3 - \lambda^2 = 0 \Rightarrow \lambda^3 + \lambda^2 = 0 \qquad \lambda^2(\lambda+1) = 0$$

$\lambda = 0$ repeated $\quad \lambda = -1$

$\lambda_{1,2} = 0$

$$\begin{vmatrix} 1-0 & 1 & 1 \\ 2 & -1-0 & 1 \\ 0 & -3 & -1-0 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 0 & -3 & -1 \end{vmatrix} \quad \text{rref} \Rightarrow \begin{vmatrix} 1 & 0 & 2/3 \\ 0 & 1 & 1/3 \\ 0 & 0 & 0 \end{vmatrix}$$

$$\begin{aligned} x_1 &= -2/3\, x_3 \\ x_2 &= -1/3\, x_3 \\ x_3 &= x_3 \end{aligned} \qquad \vec{v}_1 = \begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix}$$

Only one vector in this null space despite repeated root.

$\lambda_3 = -1$

$$\begin{vmatrix} 1+1 & 1 & 1 \\ 2 & 0 & 1 \\ 0 & -3 & 0 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 1 \\ 2 & 0 & 1 \\ 0 & -3 & 0 \end{vmatrix} \quad \text{rref} \Rightarrow \begin{vmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$\begin{aligned} x_1 &= -1/2\, x_3 \\ x_2 &= 0 \\ x_3 &= x_3 \end{aligned} \qquad \vec{v}_3 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$