Math 2568, Quiz #12, Spring 2013 Name KEY

Instructions: Show all work. You must use exact answers for all solutions.

1. Determine if the eigenspace of the matrix $\begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ spans all of $\mathbb{R}^4$. Show work and appeal to an appropriate theorem to justify your answer.

$\lambda = 1, 4, 1, 3$ 1 is repeated

$\begin{bmatrix} 1-1 & 0 & 1 & 3 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ rref $\Rightarrow$ $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ only one free variable and so only one eigenvector can be obtained for $\lambda = 1$

therefore the eigenspace has only 3 dimensions (span of 3 eigenvectors) and therefore does not span $\mathbb{R}^4$

2. Find the eigenvalues and eigenvectors of the following matrices.

a. $\begin{bmatrix} 1 & 6 \\ 2 & 5 \end{bmatrix}$ $\begin{bmatrix} 1-\lambda & 6 \\ 2 & 5-\lambda \end{bmatrix} = (1-\lambda)(5-\lambda) - 12 = 5 - 6\lambda + \lambda^2 - 12 =$

$\lambda^2 - 6\lambda - \cancel{7} = 0$ $(\lambda - 7)(\lambda + 1) = 0$ $\lambda_1 = 7,\ \lambda_2 = -1$

$\begin{bmatrix} -6 & 6 \\ 2 & -2 \end{bmatrix}$ $2x_1 = 2x_2$ $\lambda_1 = 7$ $x_1 = x_2$ $x_2 = x_2$ $\vec{v_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda_2 = 1$ $\begin{bmatrix} \cancel{3} & 6 \\ 2 & 6 \end{bmatrix}$ $2x_1 = -6x_2$ $x_1 = -3x_2$ $x_2 = x_2$ $\begin{bmatrix} -3 \\ 1 \end{bmatrix} = \vec{v_2}$

b. $\begin{bmatrix} 2 & 5 \\ 7 & 1 \end{bmatrix}$ $\begin{bmatrix} 2-\lambda & 5 \\ 7 & 1-\lambda \end{bmatrix}$ $(2-\lambda)(1-\lambda) - 35 = 2 - 3\lambda + \lambda^2 - 35 =$

$\lambda^2 - 3\lambda - 33 = 0$

$\lambda = \dfrac{3 \pm \sqrt{9 - 4(-33)}}{2} = \dfrac{3}{2} \pm \dfrac{\sqrt{141}}{2}$

$\vec{v_1}$: $\begin{bmatrix} 2 - (3/2 + \frac{\sqrt{141}}{2}) & 5 \\ 7 & 1 - (3/2 + \frac{\sqrt{141}}{2}) \end{bmatrix} = \begin{bmatrix} 1/2 - \frac{\sqrt{141}}{2} & 5 \\ 7 & -\frac{1}{2} - \frac{\sqrt{141}}{2} \end{bmatrix}$

$7x_1 = \left(\frac{1}{2} + \frac{\sqrt{141}}{2}\right) x_2 \Rightarrow$ $x_1 = \dfrac{1+\sqrt{141}}{14} x_2$

$x_2 = x_2$ $x_2 = x_2$

$\vec{v_1} = \begin{bmatrix} 1+\sqrt{141} \\ 14 \end{bmatrix}$

$\vec{v_2} = \begin{bmatrix} 1-\sqrt{141} \\ 14 \end{bmatrix}$