Math 2568, Quiz #14, Summer 2013 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round.

1. Consider the space defined by $W = span\left\{\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}\right\}$. Find $W^{\perp}$.

$$\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix} \qquad \begin{aligned} x_1 &= -x_3 \\ x_2 &= 0 \\ x_3 &= x_3 \end{aligned} \qquad \vec{b}_3 = \begin{bmatrix}-1\\0\\1\end{bmatrix} = W^{\perp}$$

2. Given $\vec{y} = \begin{bmatrix}1\\4\\-7\\8\end{bmatrix}$ and $W = span\left\{\begin{bmatrix}3\\-2\\1\\-1\end{bmatrix}, \begin{bmatrix}0\\1\\2\\0\end{bmatrix}\right\}$, find $\vec{y_{\parallel}}$ (the portion of $\vec{y}$ in W, and $\vec{y_{\perp}}$ (the portion of $\vec{y}$ in $W^{\perp}$. [Hint: You do not need to find a basis for $W^{\perp}$ to do this.]

$$\text{proj}_{\vec{v}_1}\vec{y} = \frac{3-8-7-8}{9+4+1+1}\begin{bmatrix}3\\-2\\1\\-1\end{bmatrix} = \frac{-20}{15}\begin{bmatrix}3\\-2\\1\\-1\end{bmatrix} = -\tfrac{4}{3}\begin{bmatrix}3\\-2\\1\\-1\end{bmatrix} = \begin{bmatrix}-4\\8/3\\-4/3\\4/3\end{bmatrix}$$

$$\text{proj}_{\vec{v}_2}\vec{y} = \frac{0+4-14+0}{0+1+4+0}\begin{bmatrix}0\\1\\2\\0\end{bmatrix} = \frac{-10}{5}\begin{bmatrix}0\\1\\2\\0\end{bmatrix} = -2\begin{bmatrix}0\\1\\2\\0\end{bmatrix} = \begin{bmatrix}0\\-2\\-4\\0\end{bmatrix}$$

$$\vec{y_{\parallel}} = \text{proj}_{\vec{v}_1}\vec{y} + \text{proj}_{\vec{v}_2}\vec{y} = \begin{bmatrix}-4\\8/3\\-4/3\\4/3\end{bmatrix} + \begin{bmatrix}0\\-2\\-4\\0\end{bmatrix} = \begin{bmatrix}-4\\2/3\\-16/3\\4/3\end{bmatrix}$$

$$\vec{y_{\perp}} = \vec{y} - \vec{y_{\parallel}} = \begin{bmatrix}1\\4\\-7\\8\end{bmatrix} - \begin{bmatrix}-4\\2/3\\-16/3\\4/3\end{bmatrix} = \begin{bmatrix}5\\10/3\\-5/3\\20/3\end{bmatrix}$$