Math 2568, Quiz #3, Summer 2013 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Answer all parts of each question and give answers in the specified format.

1. Determine if the sets of vectors are linearly independent, by inspection. If the set is not independent, give a subset of the vectors that is.

a. $\begin{bmatrix}1\\2\\1\end{bmatrix}, \begin{bmatrix}3\\0\\4\end{bmatrix}, \begin{bmatrix}-1\\2\\2\end{bmatrix}, \begin{bmatrix}5\\0\\1\end{bmatrix}$

not independent
too many vectors for $\mathbb{R}^3$

$\left\{\begin{bmatrix}1\\2\\1\end{bmatrix}, \begin{bmatrix}3\\0\\4\end{bmatrix}, \begin{bmatrix}5\\0\\1\end{bmatrix}\right\}$

b. $\begin{bmatrix}1\\0\\1\\-1\end{bmatrix}, \begin{bmatrix}0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}2\\3\\1\\0\end{bmatrix}$

not independent
$\vec{0}$ vector

$\left\{\begin{bmatrix}1\\0\\1\\-1\end{bmatrix}, \begin{bmatrix}2\\3\\1\\0\end{bmatrix}\right\}$

c. $\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}3\\1\end{bmatrix}$

yes independent
2 vectors not multiples

✓

d. $\begin{bmatrix}0\\1\\-2\end{bmatrix}, \begin{bmatrix}3\\2\\7\end{bmatrix}, \begin{bmatrix}0\\-3\\6\end{bmatrix}$

not independent
$-3\vec{v}_1 = \vec{v}_3$

$\left\{\begin{bmatrix}0\\1\\2\end{bmatrix}, \begin{bmatrix}3\\2\\7\end{bmatrix}\right\}$

2. Show, ***using the definition***, that any 2x2 matrix of the form $A = \begin{bmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{bmatrix}$ must be a linear transformation. (You must show all the work here, not appeal to the theorem in the textbook.)

1) $\begin{bmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} \overset{\text{matrix mult.}}{=} \begin{bmatrix}a_{11}x_1 + a_{12}x_2\\a_{21}x_1 + a_{22}x_2\end{bmatrix}$

$\begin{bmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}y_1\\y_2\end{bmatrix} = \begin{bmatrix}a_{11}y_1 + a_{12}y_2\\a_{21}y_1 + a_{12}y_2\end{bmatrix}$

vector add $\Rightarrow$ $\begin{bmatrix}a_{11}x_1 + a_{12}x_2 + a_{11}y_1 + a_{12}y_2\\a_{21}x_1 + a_{22}x_2 + a_{21}y_1 + a_{22}y_2\end{bmatrix}$

$\begin{bmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}x_1+y_1\\x_2+y_2\end{bmatrix} = \begin{bmatrix}a_{11}(x_1+y_1) + a_{12}(x_2+y_2)\\a_{21}(x_1+y_1) + a_{22}(x_2+y_2)\end{bmatrix} = \begin{bmatrix}a_{11}x_1 + a_{11}y_1 + a_{12}x_2 + a_{12}y_2\\a_{21}x_1 + a_{21}y_1 + a_{22}x_2 + a_{22}y_2\end{bmatrix}$

matrix multipl.

distributive real #

equal by commutative prop of real #'s

2) $\begin{bmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}cx_1\\cx_2\end{bmatrix} = \begin{bmatrix}ca_{11}x_1 + ca_{12}x_2\\ca_{21}x_1 + ca_{22}x_2\end{bmatrix} \overset{?}{=} c\begin{bmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} \overset{\text{matrix mult.}}{=} c\begin{bmatrix}a_{11}x_1 + a_{12}x_2\\a_{21}x_1 + a_{22}x_2\end{bmatrix}$

matrix mult

scalar mult.

$= \begin{bmatrix}ca_{11}x_1 + ca_{12}x_2\\ca_{21}x_1 + ca_{22}x_2\end{bmatrix}$ dist. of real #'s

is linear

3. Write the matrix of the linear transformation if $T\left(\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}\right) = \begin{bmatrix}x_1 + 2x_2\\0\\2x_3 + x_4\\x_2 - x_4\end{bmatrix}$.

$T(\vec{x}) = A\vec{x} = \begin{bmatrix}1 & 2 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 2 & 1\\0 & 1 & 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}$