

**Instructions:** Show all work. Use exact answers unless specifically asked to round.

1. Consider the basis for  $\mathbb{R}^4$  given by the vectors:  $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} \right\}$ . Find the

representation of the vector  $\vec{v} = \begin{bmatrix} 4 \\ 0 \\ -2 \\ 3 \end{bmatrix}$  in this basis.

$$P_B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

$$[\vec{v}]_B = P_B^{-1} \vec{v} = \begin{bmatrix} -3/2 \\ 2 \\ 3 \\ 1/2 \end{bmatrix}$$

2. Given a matrix A which you know to be  $6 \times 9$  and had 5 pivots, answer the following questions:

a. What is the rank of the matrix?

5

b. What is the dimension of Nul A?

$$9 - 5 = 4$$

c. What space are the vectors in Nul A in?

$\mathbb{R}^9$

d. Is the transformation one-to-one?

no, there are free variables

e. Is it onto?

no, only 5 not 6 pivots.

f. What is the range of A, and does it span all of that space?

$\mathbb{R}^6$ , no, only a 5-dim subspace