Math 2366, Exam #3, Summer 2014 Name KEY

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. For the relation $R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3)(3,4), (4,4)\}$, on the set $A = \{1,2,3,4\}$, determine if it is reflexive, symmetric, antisymmetric and/or transitive. Explain your reasoning. (7 points)

reflexive since (a,a) exists for all a

not symmetric since $(1,2)$ in R but not $(2,1)$

antisymmetric since only symmetric pairs are (a,a)

transitive since if (a,b) and (b,c) in R, so is (a,c).

2. For the relations $R_1 = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$ and $R_2 = \{(1,1), (1,2), (2,1), (2,2), (2,3), (3,2), (3,3), (3,4), (4,3), (4,4)\}$, find the value of the following relations. (5 points each)

a. $R_1 \cup R_2$

$\{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4), (4,3), (4,4)\}$

b. $R_1 \cap R_2$

$\{(1,2), (2,3), (3,4)\}$

c. $R_1 - R_2$

$\{(1,3), (1,4), (2,4)\}$

d. $R_1 \circ R_2$

$\{(4,4)\ (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4)\}$

e. $R_2 \oplus R_1$

$\{(1,1), (1,3), (1,4), (2,1), (2,2), (3,2), (2,4), (3,3), (4,3), (4,4)\}$

3. Consider the relation $R = \{(a, b) | a \text{ and } b \text{ are the same age}\}$. Determine if this relation is an equivalence relation. Explain your reasoning. (7 points)

yes.

reflexive since a is the same age as themselves

symmetric since if a is the same age as b, then b is the same age as b.

transitive since if a is the same age as b, and b is the same age as c, then a is the same age as c.

4. Translate the logical proposition $\neg(x \vee y) \wedge (z \vee \neg x)$ into Boolean Algebra. (5 points)

$$\overline{(x+y)}\,(z+\bar{x})$$

5. Use the table below to determine that the identity $x \oplus (y+z) = (x \oplus y) + (x \oplus z)$ holds. (12 points)

x	y	z	$y+z$	$x \oplus (y+z)$	$x \oplus y$	$x \oplus z$	$(x \oplus y)+(x \oplus z)$
1	1	1	1	0	0	0	0
1	1	0	1	0	0	1	1
1	0	1	1	0	1	0	1
1	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1
0	1	0	1	1	1	0	1
0	0	1	1	1	0	1	1
0	0	0	0	0	0	0	0

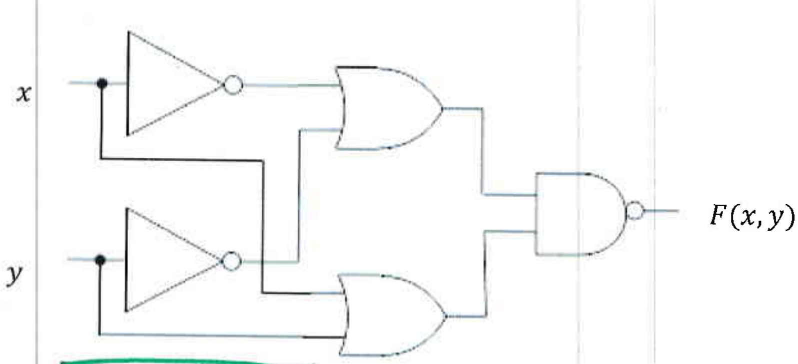
6. Find the sum-of-products expansion of $F(x, y, z) = (\bar{x} + y)\overline{(z + \bar{y})}$. You may need to use some identities to help you simplify. (10 points)

$$\bar{x}\,\overline{(z+\bar{y})} + y\,\overline{(z+\bar{y})} = \bar{x}\bar{z}y + y\bar{z}y = \bar{x}\bar{z}y + y\bar{z}$$

$$\bar{x}y\bar{z} + (x+\bar{x})y\bar{z} = \bar{x}y\bar{z} + xy\bar{z} + \bar{x}y\bar{z} =$$

$$\bar{x}y\bar{z} + xy\bar{z}$$

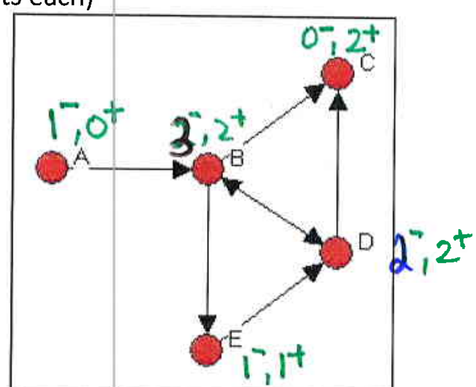
7. A logical circuit is shown below. Write the Boolean function $F(x, y)$ that is presented by the circuit. (6 points)



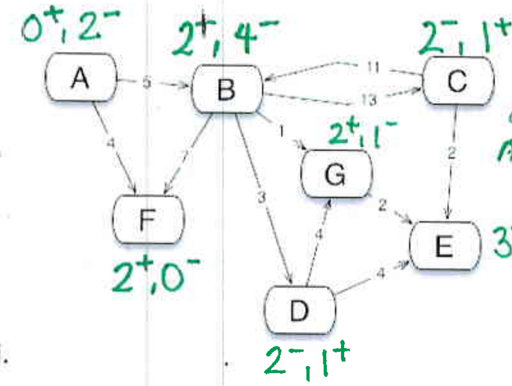
$\overline{(\bar{x}+\bar{y})(x+y)}$

8. For each of the graphs below, determine whether the graphs are a) directed, undirected, or mixed, b) a simple graph, a multigraph or a pseudograph, c) the degree of each vertex (for directed graphs, find the degree-in and the degree-out; you will need to label each vertex if they are not already labeled), d) if the graph is complete, bipartite, a wheel, a cycle, or none of these. (6 points each)

directed
Simple
none



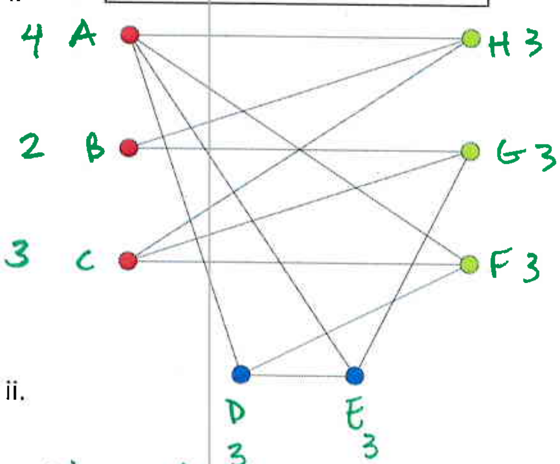
i.



directed
multigraph
none

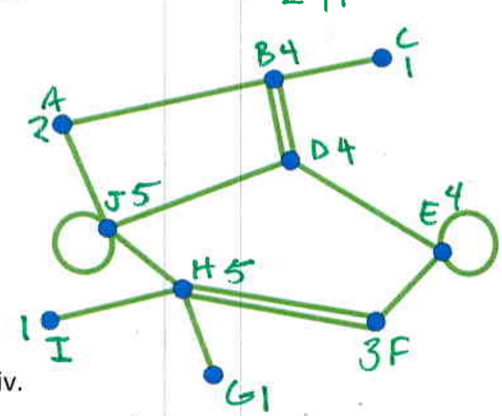
iii.

+ = in
- = out



ii.

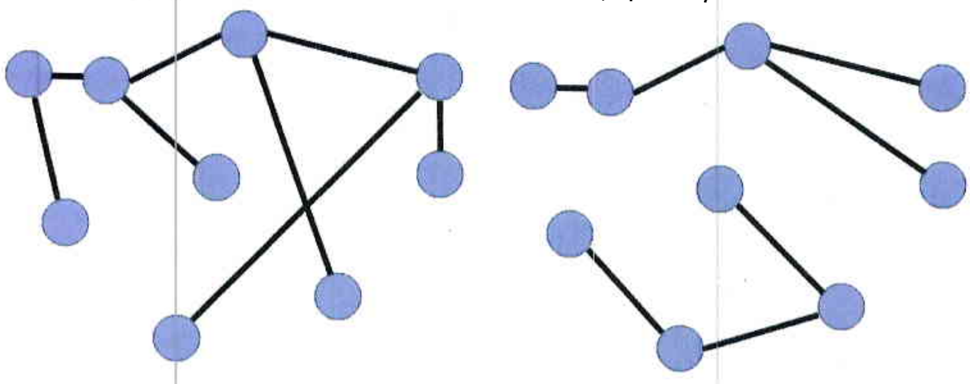
undirected
Simple
none

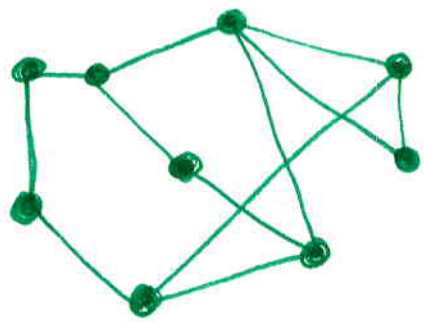


iv.

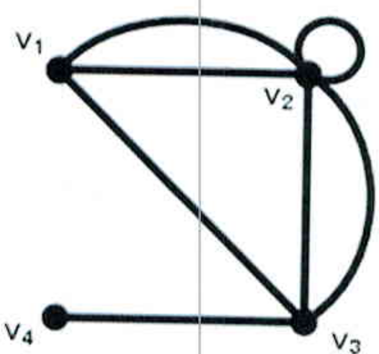
undirected
pseudograph
none

9. For the two graphs shown below, find their union. (7 points)





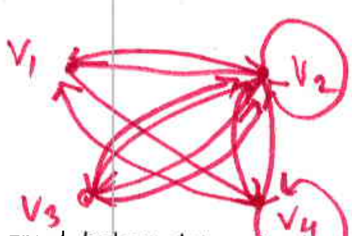
10. For the graph shown below, write an adjacency matrix. (6 points)

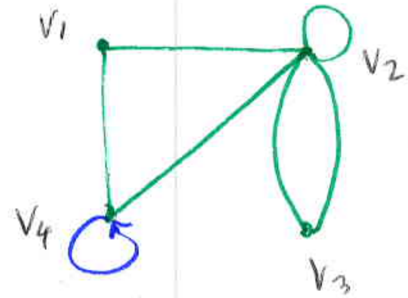


$$\begin{bmatrix} 0 & 2 & 1 & 0 \\ 2 & 2 & 2 & 0 \\ 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

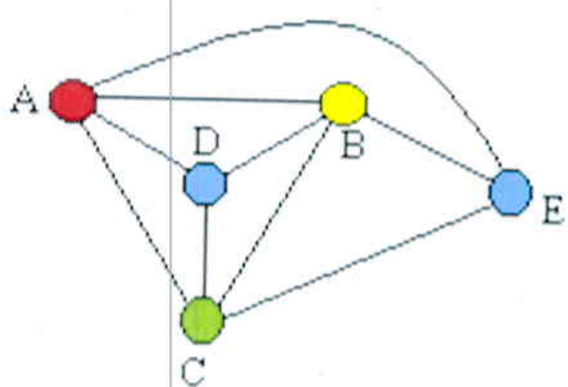
11. Using the adjacency matrix shown below, draw the graph that it represents. (6 points)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 2 & 1 \\ 0 & 2 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$





12. For the graph below, determine how many paths of length 3 are possible starting at E and ending at D. List them all. (12 points)



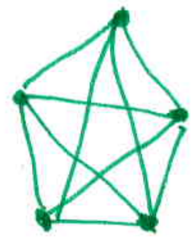
create adjacency matrix and find A^3

the a_{56} entry is 6 so there are six paths of length 3 between D & E

{E, B, D} {D, B, E}
{E, A, D} {D, A, E}
{E, C, D} {D, C, E}

13. Draw examples of graphs of each of the following types. (4 points)

a. K_5



b. C_5

c. W_5

d. Q_3