2366 Homework #2 Key

1. a. Addition b. Simplification c) modus ponens d) hypothetical syllogism

2 a. If it was sunny on Tuesday then it neither rained nor snowed. If it neither rained nor snowed, then Tuesday was not a day I took off. (Modus tollens). Therefore, Thursday was the day I took off (Disjunctive syllogism). If I took the day off on Thursday, then it either rained or snowed on Thursday. Since it didn't snow on Thursday, it must have rained. (Disjunctive Syllogism.)

b. If I am either clever or lucky and I'm not lucky, then I must be clever. (disjunctive syllogism). If I am lucky, then I will win the lottery, but since I'm not lucky, I can't tell if I'll win the lottery or not.

c. All rodents gnaw their food. Mice are rodents, therefore mice gnaw their food. (hypothetical syllogism). Rabbits do not gnaw their food, therefore rabbits are not rodents. (modus tollens). Bats are not rodents, but I can't tell whether bats gnaw their food or not.

d. All foods that are healthy to eat do not taste good. Tofu is healthy to eat, therefore it does not taste good. (hypothetical syllogism) You only eat what tastes good. You do not eat tofu. Cheeseburgers are not healthy to eat, but I can't tell if you eat it or not.

3 $\exists x\, H(x)$ implies someone is happy, but it need not be Lola. (unless the domain only includes Lola and no one else.)

4. a. this is not valid since $n=-2$ would also make $n^2>1$ and $-2 \not> 1$ this is the fallacy of affirming the conclusion.

4b. This is not valid. This is denying the hypothesis
c. also invalid. affirming the conclusion.

5. Simplification only applies to "and" and not "or"
also there is no need to "prove" both pieces to reconstruct an "or" statement

6. $n+m$ where $n = 2k+1$ and $m = 2p+1$ for some integers k & p.
then $n+m = 2k+1 + 2p+1 = 2p+2k+2 = 2(k+p+1)$ This is even
So it divides by 2 and leaves an integer $k+p+1$. Therefore the sum of 2 odd integers is always even.

7. Suppose that $3x+2$ is even. This implies that $3x+2 = 2k$ for some integer k. and that $3x = 2k-2$. Since 3 is not even, x must be. The expression $x+5$ therefore must be odd since an even # plus an odd number is odd. Since $x = 2p$ (it's even) $\Rightarrow x+5 = 2p+5 = (2p+4)+1 = 2(p+2)+1$ which is odd. If $x+5$ is odd then x is even since if $x+5 = 2n+1$ then $x = 2n-4 = 2(n-4)$ which is even. if x is even then x^2 is also even since $[2(n-4)]^2 = 4(n-4)^2 = 2[2(n-4)^2]$ and so its divisible by 2. If x^2 is even then x must be even because if x is odd, $x = 2m+1$, then $(2m+1)^2 = 4m^2+4m+1$
$= 2(2m^2+2m)+1$ which is odd. Therefore x cannot be odd.
If x is even then $x = 2t$ and so $3x+2 = 3(2t)+2 = 6t+2$
$= 2(3t+1)$ which is even. Since $A \Rightarrow B \Rightarrow C \Rightarrow A$, all these statements are equivalent.

8. Let n and $n+1$ be the pair of consecutive integers. To show that there exists at least one pair such that $n=k^2$ and $n+1 = p^3$, we can just find a single instance where it occurs for some integers k and p. There is at least one such pair for $p=2$ and $k=3$: $p^3=8$ and $k^2=9$. (There is also at least two other pairs $0^2=0$ and $1^3=1$ (or vice versa), and $(-1)^3=-1$, $0^2=0$ also works for consecutive integers 0,1 and −1,0.)

9. Consider the sets of common squares:

$1^2=1$ $2^2=4$ $3^2=9$ $4^2=16$ $5^2=25$ $6^2=36$ $7^2=49$, $8^2=64$, $9^2=81$, $10^2=100$, $11^2=121$, $12^2=144$, $13^2=169$, $14^2=196$, $15^2=225$, $16^2=256$, $17^2=289$, $18^2=324$, $19^2=361$, $20^2=400$, $21^2=441$, $22^2=484$, $23^2=529$, $24^2=576$, $25^2=625$, $26^2=676$, $27^2=729$, $28^2=784$ $29^2=841$, $30^2=900$, $31^2=961$, $32^2=1024$ $33^2=1089$, $34^2=1156$, $35^2=1225$, $36^2=1296$, $37^2=1369$, $38^2=1444$ $39^2=1521$, $40^2=1600$. Let's start by sorting these by their last digit:

1		4		9		6		5 & 0	
$1^2=1$	$9^2=81$	$2^2=4$	$8^2=64$	$3^2=9$	$7^2=49$	$4^2=16$	$6^2=36$	$5^2=25$	$10^2=100$
$11^2=121$	$19^2=361$	$12^2=144$	$18^2=324$	$13^2=169$	$17^2=289$	$14^2=196$	$16^2=256$	$15^2=225$	$20^2=400$
$21^2=441$	$29^2=841$	$22^2=484$	$28^2=784$	$23^2=529$	$27^2=729$	$24^2=576$	$26^2=676$	$25^2=625$	$30^2=900$
$31^2=961$	$39^2=1521$	$32^2=1024$	$38^2=1444$	$33^2=1089$	$37^2=1369$	$34^2=1156$	$36^2=1296$	$35^2=1225$	$40^2=1600$

In every instance where the last digit is odd the preceding digit is even, and in every case where the last digit is even, the preceding digit is even for 4 and 0 squares but odd when last digit is 6.

Let's take the odd integers first. We can write any odd integer as $(2k+1)$ for a k in $\{0,1,2,3,4\}$ plus and higher places that are all integer multiples of 10. So a general odd integer is $m \cdot 10 + 2k+1$ where m is any integer.

9 cont'd

if we square $10m+2k+1$ we get $100m^2+20mk+10m+20mk+4k^2+2k+10m+2k+1 = 100m^2+40mk+20m+4k^2+4k+1$. all the terms in this expression are even except the 1, so the #'s clearly odd.

for the given values of k we can have

$k=0$ $\quad 100m^2+20m+1$

$k=1$ $\quad 100m^2+40m+20m+4+4+1 = 100m^2+60m+9$

$k=2$ $\quad 100m^2+80m+20m+16+8+1 = 100m^2+100m+25$

$k=3$ $\quad 100m^2+120m+20m+36+12+1 = 100m^2+120m+49$

$k=4$ $\quad 100m^2+160m+20m+64+16+1 = 100m^2+180m+81$

all the last digits are odd, but what remains are even powers of 10.

explicitly for $k=0$ $\quad 100m^2+20m+1 = 10(10m^2+2m)+1$

$k=1$ $\quad 100m^2+60m+9 = 10(10m^2+6m)+9$

$k=2$ $\quad 100m^2+100m+25 = 10(10m^2+10m+2)+5$

$k=3$ $\quad 100m^2+120m+49 = 10(10m^2+12m+4)+9$

$k=4$ $\quad 100m^2+180m+81 = 10(10m^2+18m+8)+1$

thus the last digit is odd and all tens digits are even.

Consider now the even integers. So 2k for $\{0,1,2,3,4\}$ in their last digits and 10m for all other digits in the # for some positive integer. So any number equivalent to $10m+2k$. The square becomes $100m^2+40mk+4k^2$. This is even since its divisible by 2, i.e. $2(50m^2+20mk+2k^2)$.

let's consider our possible values for k:

$k=1$ $\quad 100m^2+40m+4 = 10(10m^2+4m)+4$

$k=2$ $\quad 100m^2+80m+16 = 10(10m^2+8m+1)+6$

$k=3$ $\quad 100m^2+120m+36 = 10(10m^2+12m+3)+6$

$k=4$ $\quad 100m^2+160m+64 = 10(10m^2+16m+6)+4$

$k=0$ $\quad 100m^2 = 10(10m^2)$

even squares end in only 4, 6, 0. Both final digits are even for 4 and 0 final, but are odd 2nd digits for squares ending in 6.

9 cont'd.
These calculations also show that perfect squares cannot end in 2, 3, 7 or 8.

10. a. $\{1, -1\}$

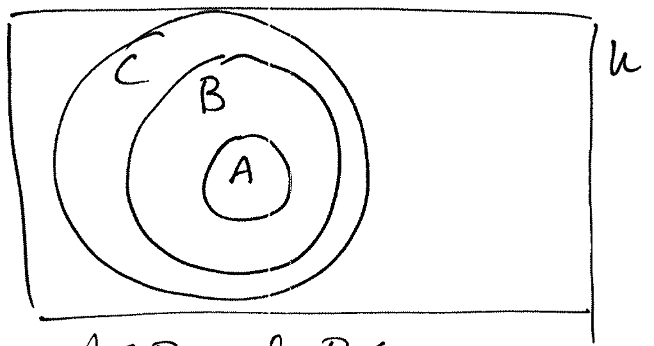
b. $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

c. $\{0, 1, 4, 9, 16, 25, 36, 49, 64, 81\}$

d. $\emptyset$ or $\{\ \}$

11. a. true b. false, c. false, d. true, e. true, f. true, g. true

12.



$A \subset B$ and $B \subset C$

13. a. $A \times B = \{(a,y), (a,z), (b,y), (b,z), (c,y), (c,z), (d,y), (d,z)\}$ $|A \times B| = 8$

b. $A \times B \times C = \{(a,y,0), (a,y,1), (a,z,0), (a,z,1), (b,y,0), (b,y,1), (b,z,0), (b,z,1), (c,y,0), (c,y,1), (c,z,1), (c,z,0), (d,y,0), (d,y,1), (d,z,0), (d,z,1)\}$ $|A \times B \times C| = 16$

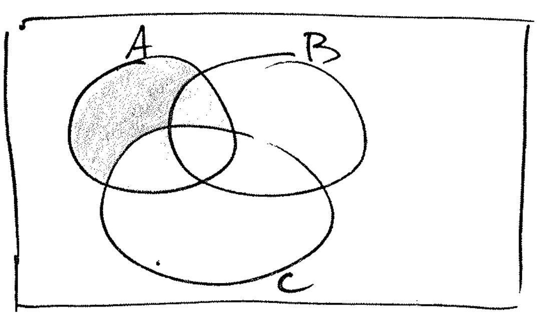
c. $A^2 = A \times A = \{(a,a), (a,b), (a,c), (a,d), (b,a), (b,b), (b,c), (b,d), (c,a), (c,b), (c,c), (c,d), (d,a), (d,b), (d,c), (d,d)\}$

14. a. $A \cup B = \{0, 1, 2, 3, 4, 5, 6\}$

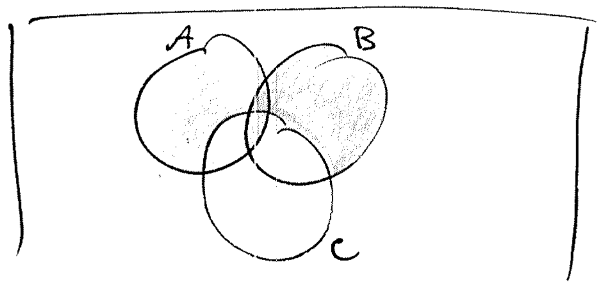
b. $A \cap B = \{3\}$

c. $A - B = \{1, 2, 4, 5\}$

15. $(A \cap \overline{B}) \cup (A \cap \overline{C})$



$A \cup (B - C)$



16\. a. $A \cup B = A$ Then B is a subset of A $B \subset A$

b. $A - B = A$ Then $A \cap B = \emptyset$ A & B are disjoint.

17\. a. $\{3,4,5\}$ = 00011100000

b. $\{1,3,6,10\}$ = 01010010001

c. 0011 1100 111 = $\{2,3,4,5,8,9,10\}$

d. 0010 0000 001 = $\{2,10\}$