2366 Homework #3 Key

1\. a. $f(x) = 1/x$ it is not defined at 0 in $\mathbb{R}$ nor is there any x value that maps onto 0 in the range.

b. $f(x) = \sqrt{x}$ is not defined for $x < 0$ and can't map onto any values in the range < 0 either.

c. $f(x) = \pm\sqrt{x^2+1}$ this maps all the $x \in \mathbb{R}$ onto something. but it's both not a function since each x maps onto 2 values, but also it misses all the values in the range set $\mathbb{R}$ between $(-1,1)$.

2\. a. D: $\mathbb{Z}^+$, R: $\{0,1,2,3,4,5,6,7,8,9\}$

b. D: all bit strings R: $\mathbb{N}$

c. $\mathbb{Z}^+$: D , R: $\{1,4,9,16,25,\ldots\}$

d. D: $\mathbb{Z}^+$, R: $\{1,2,3,4,5,6,7,8,9\}$

e. D: $\mathbb{Z}^+ \times \mathbb{Z}^+$, R: $\mathbb{Z}^+$

3\. a. not one-to-one or onto

b. not one-to-one, but is onto

c. not one-to-one & not (necessarily) onto
more than one person could get an A, not necessarily every grade will be used.

d. not one-to-one or onto (misses -2 in the domain, and 1 in the range)

e. bijection

4\. $f(n) = n+3$ (misses 1,2,3 in range)

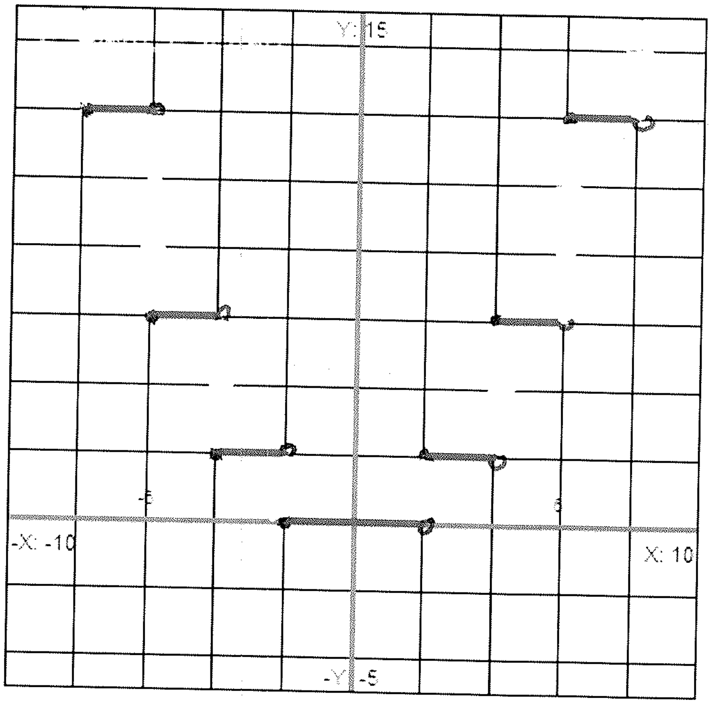
5\. a. See attached graphs

b. see attached graphs

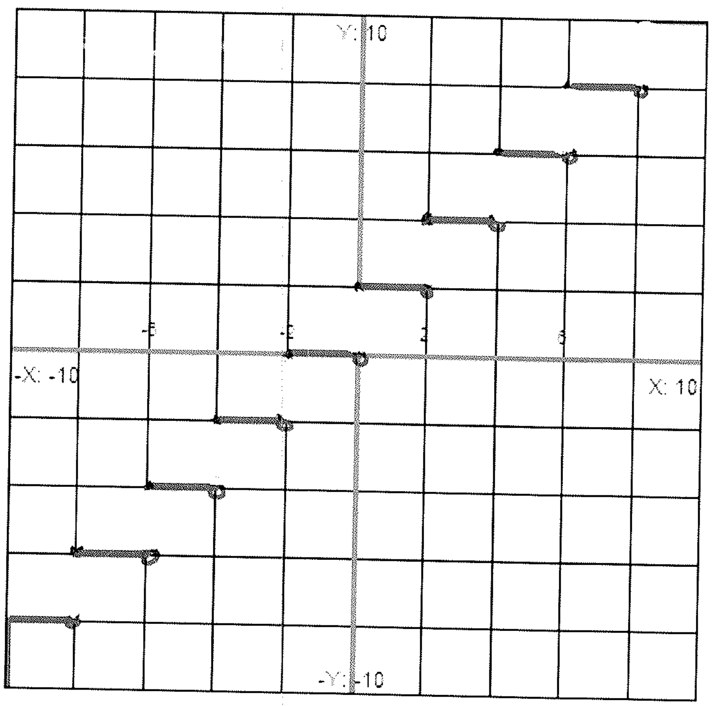
6a. 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, ...

b. 1, 1, 3, 3, 5, 5, 7, 7, 9, 9, ...

5. a.



b.



6.c. 2, 4, 6, 10, 16, 26, 42, 68, 110, 178, ...

d. 1, 0, -2, -3, 8, 95, 684, 4991, 40,256, 362,799, ...

e. 1, 2, 2, 2, 2, 3, 3, 3, 3, 3, ...

7 a. 1, 2, 5, 16, 31, 79, 172, 409, 925, 2153, ...

b. 1, 2, 0, 1, 3, 3, 4, 7, 10, 14, ...

c. 1, 0, 3, 2, 5, 4, 7, 6, 9, 8, ...

d. 1, 0, 16, 31, 80, 161, 342, 687, ...

8.a. one one, one zero, then 2 ones, then 2 zeros, then 3 ones, then 3 zeros, etc.

b. $15 - 7n = a_n$ starting at $n=0$

c. $a_n = n! + 1$ starting at $n=1$

d. $a_n = n^3 - 1$ starting at $n=1$

e. $2^{2^n} = a_n$ starting at $n=1$

9.a. $\sum_{j=0}^{4} (-2)^j = (-2)^0 + (-2)^1 + (-2)^2 + (-2)^3 + (-2)^4 = 1 - 2 + 4 - 8 + 16 = 11$

b. $\sum_{j \in S} \left(\frac{1}{j}\right)$, $S = \{1, 3, 5, 7\} = \frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} = \frac{176}{105}$

c. $\sum_{i=1}^{2} \sum_{j=1}^{3} (i+j) = \sum_{i=1}^{2} [(i+1) + (i+2) + (i+3)] = \sum_{i=1}^{2} 3i+6 = 3+6+6+6 = 21$

d. $\sum_{i=1}^{3} \sum_{j=1}^{4} (i^2 j) = \sum_{i=1}^{3} (i^2 + 2i^2 + 3i^2 + 4i^2) = \sum_{i=1}^{3} 10i^2 = 10(1+4+9) = 140$

e. $\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \dots = \frac{n-1}{n}$ telescoping series

$S_n = \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots = 1 - \frac{1}{n+1}$

f. $\sum_{k=1}^{n} (2k-1) = 2\sum_{k=1}^{n} k - \sum_{k=1}^{n} 1 = 2 \cdot \frac{n(n+1)}{2} - n = n^2 + n - n = n^2$

0.a. Countably infinite $f(n) = -n$

b. countably infinite $f(n) = 100 - n$

c. Countably infinite $f(n) = \begin{cases} 7 \cdot \frac{n}{2} & n \text{ is even} \\ -7\left(\frac{n-1}{2}\right) & n \text{ is odd} \end{cases}$

6 d. countably infinite $f(n) = -2n+1$

e. countably infinite $f(n) = \begin{cases} \text{even } n \text{ onto } \left(\frac{n}{2}, 2\right) \\ \text{odd } n \text{ onto } \left(\frac{n+1}{2}, 3\right) \end{cases}$

f. countably infinite

$$\begin{array}{ccccc} \frac{1}{5} & \frac{2}{5} & \frac{3}{5} & \frac{4}{5} & \cdots \\ \frac{1}{6} & \cancel{\frac{2}{6}} & \cancel{\frac{3}{6}} & \frac{4}{6} & \cdots \\ \frac{1}{7} & \frac{2}{7} & \frac{3}{7} & \frac{4}{7} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \end{array}$$

etc.

skipping anything that can be reduced

g. countably infinite map in order: $\{1, 2, 4, 5, 7, 8, 10, 11, \text{etc} \ldots\}$ for all even integers and for odd $\{-1, -2, -4, -5, -7, -8, \ldots\}$

h: countably infinite map in order the decimals with n # of 1's positive for n even and negative for n odd

i. countably infinite map like rationals

$$\begin{array}{ccccc} (1,1) & (1,2) & (1,3) & (1,4) & \ldots \\ (2,1) & (2,2) & (2,3) & (2,4) & \ldots \\ (3,1) & (3,2) & (3,3) & (3,4) & \ldots \\ (4,1) & (4,2) & (4,3) & (4,4) & \ldots \\ \vdots & \vdots & \vdots & \vdots & \end{array}$$

etc.

11. a. $A = \mathbb{R}$ $\quad B = \mathbb{R} - \{0\}$ $\quad A - B = \{0\}$

b. $A = \mathbb{R}$ $\quad B =$ Irrationals $\quad A - B = \mathbb{Q}$

c. $A = \mathbb{R}$ $\quad B = (0, 1)$ $\quad A - B = (-\infty, 0) \cup (1, \infty)$

12. $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ for $n = 1$, the base case $\displaystyle\sum_{i=1}^{1} i^2 = 1^2 = 1$

and $\frac{1(2)(3)}{6} = 1$ $\quad 1 = 1$ so this is true. Now suppose its true for n and we need to show it holds for $n+1$. Consider

$$\sum_{i=1}^{n+1} i^2 = \sum_{i=1}^{n} i^2 + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2 = \frac{n(n+1)(2n+1) + 6(n+1)^2}{6}$$

$$= \frac{(n+1)[n(2n+1) + 6(n+1)]}{6} = \frac{(n+1)[2n^2 + n + 6n + 6]}{6} = \frac{(n+1)(2n^2 + 7n + 6)}{6}$$

$$= \frac{(n+1)(2n+3)(n+2)}{6} = \frac{(n+1)(n+2)(2n+3)}{6} = \frac{[n+1][(n+1)+1][2(n+1)+1]}{6}$$

15.c. $f(0)=0 \quad f(n+1)=f(n)+5$ (5)

d. $f(0)=2 \quad f(n+1)=\begin{cases} f(n-1)+3 & \text{for } n \text{ even} \\ f(n-1)-3 & \text{for } n \text{ odd} \end{cases}$

$f(1)=-1$

16.a. if $F(1)=1$, $F(2)=1+F(\lfloor 1 \rfloor)=1+F(1)=2$

$F(3)=1+F(\lfloor \frac{3}{2} \rfloor)=1+F(1)=2 \quad F(4)=1+F(\lfloor \frac{4}{2} \rfloor)=1+F(2)=3$

but $F(1)$ is defined to be 1, but also $1+F(0)$ but we don't know what $F(0)$ is. This would be okay if we said $n>1$ instead of $n \geq 1$.

b. $F(1)=1 \quad F(2)=1+F(\frac{2}{2})=1+F(1)=2 \quad F(3)=F(9-1)=F(8)$

But that doesn't mean anything sequentially since we don't yet know what $F(8)$ is in the sequence.