2366 Homework #4 Key

1a. $18 \times 325 = 5850$

$\binom{18}{2}\binom{325}{2} = 8{,}055{,}450$

b. $26^3 = 17{,}576$

c. $26^4 - 25^4 = 66351$

all possible ← no x's

d. $\underline{4}\ \underline{4}\ \underline{4}\ \underline{4}\ \underline{A} = 4^4(1) = 256$

e. $142 - 14 = 128$

less than 1000   less than 100

f. 142 divisible by 7
90 divisible by 11
12 divisible by both

220

$999 - 220 = 779$ divisible by neither

g. $\underline{\ }\ \underline{\ }\ \underline{10}$ with 4 4 $= 10 \times (3)$ where the non-4 can appear

$= 30$

h. $26^3 10^3 + 10^3 26^3 =$

35,152,000

i. $_{26}P_8 = 6.299 \times 10^{10}$

j. $26^8 - 21^8 = 1.71 \times 10^{11}$

all cases   no vowels

k. $4^{10} = 1{,}048{,}576$ or

one-to-one $_{10}P_4 = 5040$

l. $\frac{5!}{2} = 60$

m. $\underline{1}\ \underline{1}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2} = 2^4 \cdot 2 + 2^3 \cdot 3 = 56 \times 2$

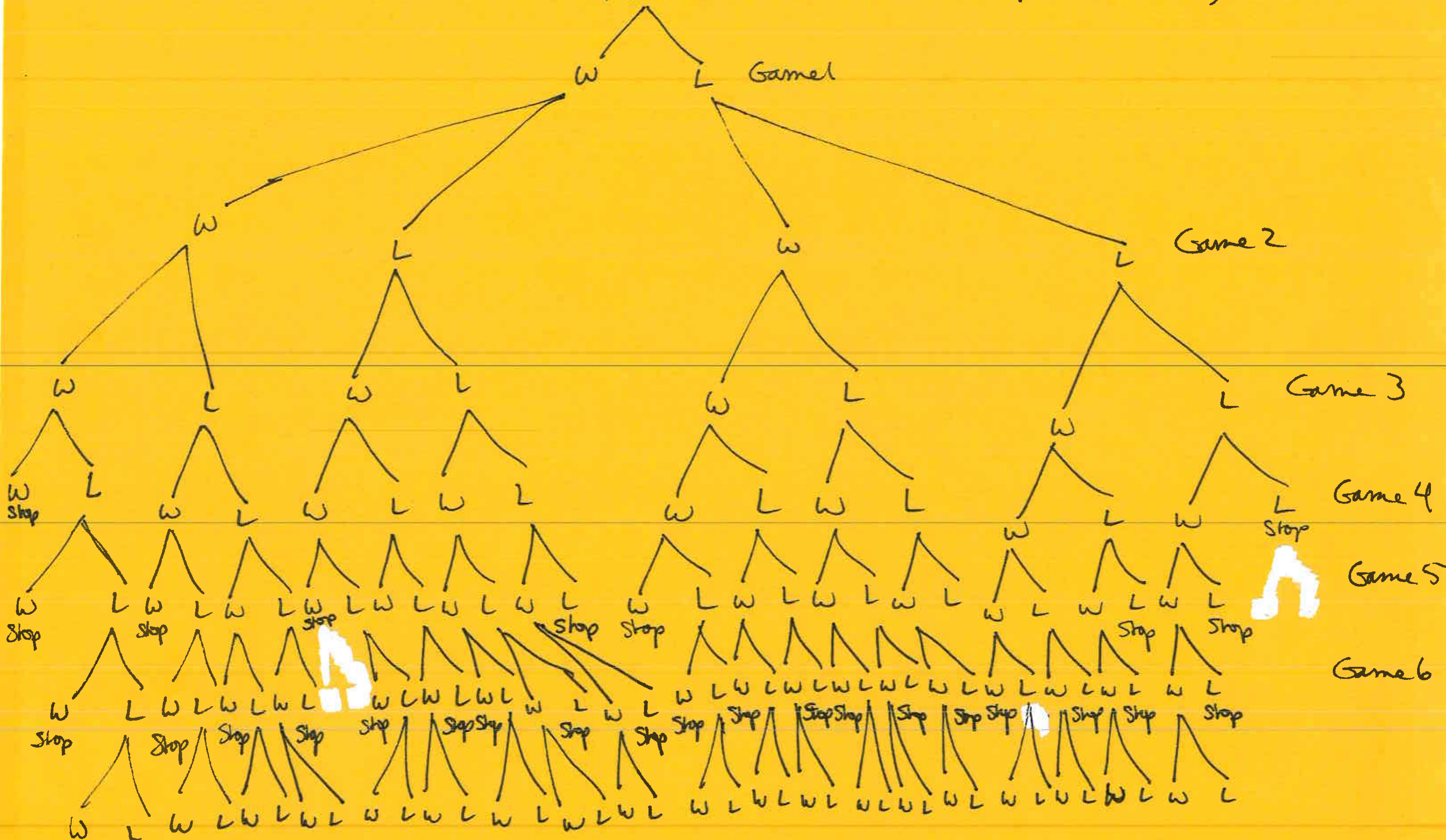
$= 112$

$\underline{1}\ \underline{1}\ \underline{1}\ \underline{2}\ \underline{2}\ \underline{2}$

this is strictly 5 1's or 0's and not at least 5's 1's or 0's

(2) i.n.

Team A (Team B is same w/ W & L switched)



38+19+7+2 = 66 ways

2. 3, 13

3. n consecutive integers

$k, k+1, k+2, k+3, k+4 + \cdots k+n-1$

we use modulo division (associating a # w/ it's remainder after division by an integer. $0 = k \bmod n$ = There exists p such that $k = np$

$3 = k \bmod n$ = there exists p such that $k = np+3$. we divide up the modulos into n boxes. regardless of where we start, $(k+1) \bmod n$ goes into the box immediately after $k \bmod n$. using the fact that $(k+n) \bmod n = k \bmod n + n \bmod n = k \bmod n$ (since n divides evenly into itself) by the time we get to $k+n-1$, we will be in the box immediately before $k \bmod n$, which means at least one of the integers will have had to have been $(k+l) \bmod n = 0$.

4. if we divide the class into 2 "boxes" male & female, the most equally divided class will have the first 8 students exactly 4 female and 4 male. The 9th student will have to mean there are at least 5 of one gender by the pigeonhole principle.

5. $1920 = 100 \times 19 + 20 = 1920$

6. 3-Permutations <u>123</u> 132 213 231 312 321 <u>124</u> 142 214 241 412 421 <u>125</u> 152 215 251 512 521 <u>234</u> 243 324 342 423 432 <u>235</u> 253 325 352 523 532 <u>345</u> 354 435 453 534 543 <u>134</u> 143 314 341 413 431 <u>135</u> 153 315 351 531 513 <u>145</u> 154 415 451 514 541 <u>245</u> 254 425 452 542 524

Underlined are 3-combinations

7a. $6! = 720$

b. $2^8 = 256$ $256 - 1 - 8 - 28 = 219$

c. CDE $5! * 6! - ! - ! - ! - ! - ! = 6! = 720$

7d. $\binom{25}{4} = 12,650$ $P(25,4) = 303,600$

e. 4, 2 5, 1 6, 0

$\binom{15}{4}\binom{10}{2} + \binom{15}{5}\binom{10}{1} + \binom{15}{6}\binom{10}{0} = 61,425 + 30,030 + 5005 = 96,460$

8.a. $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

b. $(3x)^5 + 5(3x)^4(2y) + 10(3x)^3(2y)^2 + 10(3x)^2(2y)^3 + 5(3x)(2y)^4 + (2y)^5 =$

$243x^5 + 810x^4y + 1080x^3y^2 + 720x^2y^3 + 240xy^4 + 32y^5$

c. $-(2)^{10}(x^9)\binom{19}{9} = -94,595,072$

d. $x^{101}y^{99}$ $\binom{200}{101}(2x)^{101}(-3y)^{99} = -\binom{200}{101}2^{101}3^{99}$ (calc overflow)

e. $\binom{100}{i}(x^2)^{100-i}(x^{-1})^i = \binom{100}{i}x^{2n-2i-i} = \binom{100}{i}x^{2n-3i}$

$k = 2n-3i = 200-3i \Rightarrow 3i = 200-k \Rightarrow i = \left(\frac{200-k}{3}\right)$

$\binom{100}{\frac{200-k}{3}}x^k$ where combination formula is defined.

f. $((x+y)+z)^{10} = (x+y)^{10} + 10(x+y)^9z + 45(x+y)^8z^2 + 120(x+y)^7z^3 +$
$210(x+y)^6z^4 + 252(x+y)^5z^5 + 210(x+y)^4z^6 + 120(x+y)^3z^7 +$
$45(x+y)^2z^8 + 10(x+y)z^9 + z^{10} =$

$x^{10} + 10x^9y + 45x^8y^2 + 120x^7y^3 + 210x^6y^4 + 252x^5y^5 + 210x^4y^6 + 120x^3y^7$
$+ 45(x^2y^8) + 10xy^9 + y^9 + 10(x^9 + 9x^8y + 36x^7y^2 + 84x^6y^3 + 126x^5y^4$
$+ 126x^4y^5 + 84x^3y^6 + 36x^2y^7 + 9xy^8 + y^9)z + 45(x^8 + 8x^7y + 28x^6y^2$
$+ 56x^5y^3 + 70x^4y^4 + 56x^3y^5 + 28x^2y^6 + 8yx^7 + x^7)z^2 + 120(x^7 + 7x^6y +$
$21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7)z^3 + 210(x^6 + 6x^5y +$
$15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6)z^4 + 252(x^5 + 5x^4y + 10x^3y^2 +$
$10x^2y^3 + 5xy^4 + y^5)z^5 + 210(x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4)z^6 + 120(x^3 +$
$3x^2y + 3xy^2 + y^3)z^7 + 45(x^2 + 2xy + y^2)z^8 + 10(x+y)z^9 + z^{10} =$

8 cont'd

$x^{10} + 10x^9y + 10x^9z + 45x^8y^2 + 90x^8yz + 45x^8z^2 + 120x^7y^3 + 360x^7y^2z$
$+ 360x^7yz^2 + 120x^7z^3 + 210x^6y^4 + 840x^6y^3z + 1260x^6y^2z^2 + 840x^6yz^3 +$
$+ 210x^6z^4 + 252x^5y^5 + 1260x^5y^4z + 2520x^5y^3z^2 + 2520x^5y^2z^3 + 1260x^5yz^4$
$+ + 252x^5z^5 + 210x^4y^6 + 1260x^4y^5z + 3150x^4y^4z^2 + 4200x^4y^3z^3 +$
$3150x^4y^2z^4 + 1260x^4yz^5 + 210x^4z^6 + 120x^3y^7 + 840x^3y^6z + 2520x^3y^5z^2$
$+ 4200x^3y^4z^3 + 4200x^3y^3z^4 + 2520x^3y^2z^5 + 840x^3yz^6 + 120x^3z^7 +$
$45x^2y^8 + 360x^2y^7z + 1260x^2y^6z^2 + 2520x^2y^5z^3 + 3150x^2y^4z^4 + 2520x^2y^3z^5$
$+ 1260x^2y^2z^6 + 360x^2yz^7 + 45x^2z^8 + 10xy^9 + 90xy^8z + 360xy^7z^2 +$
$840xy^6z^3 + 1260xy^5z^4 + 1260xy^4z^5 + 840xy^3z^6 + 360xy^2z^7 + 90xyz^8$
$+10xz^{10} + y^{10} + 10y^9z + 45y^8z^2 + 120y^7z^3 + 210y^6z^4 + 252y^5z^5 + 210y^4z^6 +$
$120y^3z^7 + 45y^2z^8 + 10yz^9 + z^{10}$

9. $\binom{2n}{n+1} + \binom{2n}{n} = \dfrac{(2n!)}{(n+1)!(2n-n-1)!} + \dfrac{(2n)!}{n!(2n-n)!} = \dfrac{(2n)!}{(n+1)!(n-1)!} + \dfrac{2n!}{n!n!} = \dfrac{2n!}{n!(n+1)(n-1)!} + \dfrac{2n!}{n!(n-1)!n}$

$\frac{1}{2}\binom{2n+2}{n+1} = \frac{1}{2}\left(\dfrac{(2n+2)!}{(n+1)!(2n+2-n-1)!}\right) = \frac{1}{2}\left(\dfrac{(2n+2)!}{(n+1)!(n+1)!}\right) = \frac{1}{2}\left(\dfrac{(2n)!(2n+1)(2n+2)}{(n+1)!(n+1)!}\right)$

$\dfrac{2n!}{n!(n-1)!}\left(\dfrac{1}{n+1} + \dfrac{1}{n}\right) = \dfrac{2n!}{n!(n-1)!}\left(\dfrac{n+n+1}{(n+1)n}\right) = \dfrac{2n!(2n+1)}{n!(n-1)!(n+1)n} =$

$\dfrac{(2n)!(2n+1)}{(n+1)!(n-1)!n} = \dfrac{(2n)!(2n+1)}{(n+1)!n!}\dfrac{2}{2}\cdot\dfrac{(n+1)}{(n+1)} = \dfrac{(2n)!(2n+1)(2n+2)}{2(n+1)!(n+1)!} = \dfrac{1}{2}\dfrac{(2n+2)!}{(n+1)!(n+1)!} = \dfrac{1}{2}\binom{2n+2}{n+1}$

10. a. $(3^5) = 243$ b. $\binom{5+5-1}{5-1} = \binom{9}{4} = 126$ c. $\binom{20+5-1}{19} = \binom{24}{19} = 42{,}504$

d. $\dfrac{14!}{3!3!2!2!2!} = 302{,}702{,}400$ e. $\dfrac{11!}{1!4!4!2!} = 34{,}650$

f. $\binom{52}{7}\binom{45}{7}\binom{38}{7}\binom{31}{7}\binom{24}{7} = 6.97 \times 10^{34}$ if the players are treated as distinguishable

g. $\square$ $\square$ $\square$ 2 2 1   3 1 1   indistinguisable = 2

distinguishable 221, 212, 122, 311, 131, 113   6
boxes indistinguishable balls

labeled balls $\binom{5}{3}\binom{2}{1}\binom{1}{1} + \binom{5}{2}\binom{3}{2}\binom{1}{1} =$
$20 + 30 = 50$