Math 2366 Homework #6 Key

(3g)

1.a. $\{(0,0), (1,1), (2,2), (3,3)\}$ reflexive, symmetric, antisymmetric, transitive

b. $\{(1,3), (2,2), (3,1), (4,0)\}$ none

c. $\{(1,0), (2,0), (2,1), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2), (4,3)\}$ none

d. $\{(1,1), (1,2), (2,1), (1,3), (3,1), (2,3), (3,2), (4,1), (4,3)\}$ not symmetric $(0,1)$ $(1,0)$? not reflexive, not transitive

e. $\{(1,1), (1,2), (1,3), (1,0), (2,0), (2,2), (3,0), (3,3)\}$ not reflexive, not symmetric, antisymmetric, not transitive

f. $\{(1,2), (2,1)\}$ symmetric, not reflexive, not transitive

2a. not symmetric, not anti-symmetric, not reflexive, not transitive
not irreflexive, not asymmetric

b. symmetric, reflexive, transitive, not irreflexive, not asymmetric or anti-symmetric

c. not reflexive, not anti-symmetric, symmetric, not transitive,
is irreflexive, not asymmetric

d. not reflexive, not symmetric, not anti symmetric, not transitive,
irreflexive, asymmetric.

e. reflexive, antisymmetric, transitive, not irreflexive, not asymmetric
symmetric

f. irreflexive, not reflexive, not symmetric, not antisymmetric, not asymmetric,
not transitive

3a. if (x,y) in R, then $x+y=0$ (x,x) not in R since $x+x=0$ only if $x=0$; (y,x) in R since if $x+y=0$, $y+x=0$ if $x+y=0$ & $y+z=0$ then does $x+z=0$? no, unless $x=y=z=0$ Symmetric, but not reflexive or transitive

b. $xy \geq 0$ $xx \geq 0$? yes reflexive; $xy \geq 0$ $yx \geq 0$? yes so (x,y) & (y,x) in R so symmetric. if $xy \geq 0$ & $yz \geq 0$ is $xz \geq 0$? not necessarily since if $(3)(0) \geq 0$ & $(0)(-5) \geq 0$ $(3)(-5) \not\geq 0$
not transitive

3c. $x=1, y=1$ $R: \{(1,1)\}$ reflexive, symmetric & transitive

d. not reflexive since (x,x) not in set since $x \neq x$ is false. symmetric since if $x \neq y$ then $y \neq x$. not transitive since if $x \neq y$ & $y \neq z$ it does not guarantee that $x \neq z$ (see symmetry above).

e. symmetric, reflexive & transitive $(0,0)$ since $0 = 0 \bmod 7$, also any # $7k = 7n \bmod 7$. even if $n=k$. symmetric since $7k = 7n \bmod 7$, & $7n = 7k \bmod 7$. and transitive since $7k = 7n \bmod 7$ & $7n = 7q \bmod 7 \Rightarrow 7k = 7q \bmod 7$ since all are in the same equivalence class. (I've used $0 \bmod 7$ as the class, but without loss of generalization, we can make same argument for any of the 7 equivalence classes $7k, 7k+1, 7k+2, 7k+3, 7k+4, 7k+5, 7k+6$.

f. only reflexive pairs are $(0,0)$ & $(1,1)$ not symmetric since if $x = y^2$ only for 0, & 1 is $y = x^2$. not transitive. for instance $(4,2)$ & $(16,4)$ does not $\Rightarrow$ $(16,2)$ in set.

4. R^{-1} is states (b,a) where state b borders state a thus $R^{-1} = R$. $\overline{R}$ is the set of points where state a does <u>not</u> border state b.

5. consider the equivalence class $0 \bmod 3$ and $0 \bmod 4$. (similar argument can be made for any similar class).

a. $R_1 \cup R_2 = \{(0,0), (0,3), (0,6), (3,0), (3,3), (3,6), (0,4), (0,8), (4,4), (4,8)$
$(8,4), (8,12)$ $(12,12), \ldots$ etc$\}$

b. $R_1 - R_2$ the pairs that have to be removed from R_1 are anything also in R_2. in this case the removed points would be $0 \bmod 12$ pairs $(0,0), (12,12), (0,12), (12,0)$ etc.

c. $R_1 \cap R_2$ the set of points removed from $R_1 - R_2$ i.e. $0 \bmod 12$

d. $R_1 \oplus R_2$ this is the same as $R_1 \cup R_2 - R_1 \cap R_2$

6. 1a, 2b, 3c, 3e

7. {4 credit hour course}, {5 credit courses}, {6 credit courses}, {3, credit courses} {2 credit courses}, {1 credit courses} ...
   Courses a, b w/ The same # of credit hours.

   OR courses a, b offered by the same department {Math}, {English}, etc.
   there are other options also.

8. a. partition
   b. not a partition 0 used twice
   c. partition
   d. not a partition 0 not used at all.

9. a. partition
   b. partition
   c. not a partition (overlap)
   d. not a partition (integers omitted)
   e. partition
   f. partition

10. a. $1 \cdot \overline{0} = 1 \cdot 1 = 1$
    b. $1 + \overline{1} = 1 + 0 = 1$
    c. $\overline{0} \cdot 0 = 1 \cdot 0 = 0$
    d. $\overline{(1+0)} = \overline{1} = 0$

11. a. $F(x, y, z)$

x	y	$\overline{x}$	$\overline{x}y$
1	1	0	0
0	1	1	1
1	0	0	0
0	0	1	0

b. $F(x, y, z) =$

x	y	z	$\overline{y}$	$x\overline{y}$	xy	xyz	$\overline{xyz}$	$x\overline{y} + \overline{xyz}$
1	1	1	0	0	1	1	0	0
1	1	0	0	0	1	0	1	1
1	0	1	1	1	0	0	1	1
1	0	0	1	1	0	0	1	1
0	1	1	0	0	0	0	1	1
0	1	0	0	0	0	0	1	1
0	0	1	1	0	0	0	1	1
0	0	0	1	0	0	0	1	1

11. c. $F(x,y,z) = x(yz + \bar{y}\bar{z})$

x	y	z	yz	$\bar{y}$	$\bar{z}$	$\bar{y}\bar{z}$	$yz+\bar{y}\bar{z}$	$x(yz+\bar{y}\bar{z})$
1	1	1	1	0	0	0	1	1
1	1	0	0	0	1	0	0	0
1	0	1	0	1	0	0	0	0
1	0	0	0	1	1	1	1	1
0	1	1	1	0	0	0	1	0
0	1	0	0	0	1	0	0	0
0	0	1	0	1	0	0	0	0
0	0	0	0	1	1	1	1	0

12.

x	y	z	$y+z$	$x\oplus(y+z)$	$x\oplus y$	$x\oplus z$	$(x\oplus y)+(x\oplus z)$
1	1	1	1	0	0	0	0
1	1	0	1	0	0	1	1
1	0	1	1	0	1	0	1
1	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1
0	1	0	1	1	1	0	1
0	0	1	1	1	0	1	1
0	0	0	0	0	0	0	0

not identical
therefore, not an identity

13. $F(x,y,z) = (x+z)y$

$= xy + yz = xy(z+\bar{z}) + (x+\bar{x})yz =$

$xyz + xy\bar{z} + \cancel{xyz} + \bar{x}yz = xyz + xy\bar{z} + \bar{x}yz$

$F(x,y,z) = x\bar{y}$

$x\bar{y}(z+\bar{z}) = x\bar{y}z + x\bar{y}\bar{z}$

14. $\overline{x+\bar{y}} = \bar{x}y$ by de Morgan's law

$x+\bar{y}(\bar{x}+z) = x+\bar{y}\,\overline{\overline{(\bar{x}+z)}} = x+\bar{y}\,\overline{(x\bar{z})} = \overline{\overline{x+\bar{y}\,\overline{(x\bar{z})}}} =$

$\overline{\bar{x}(\overline{\bar{y}\,\overline{(x\bar{z})}})}$

15. a. $AB + \overline{AC}$

15b. $\overline{AB + \overline{C}}$

c. $(AB + AC)B$

d. $\overline{AB(CD + \overline{E})} + E$

e. $(A+B)\overline{B}$

f. $\overline{A+B} + BC$

g. $\left(\overline{\overline{X}\,\overline{Y}}\right)\overline{(XY)}$

h. $(AB)\overline{\overline{(AC)}} \oplus (B\overline{AC})$

i. $\overline{\overline{(I + J)}(J\overline{K})}$

j. $\overline{\overline{AB} \oplus (A \oplus B)}$