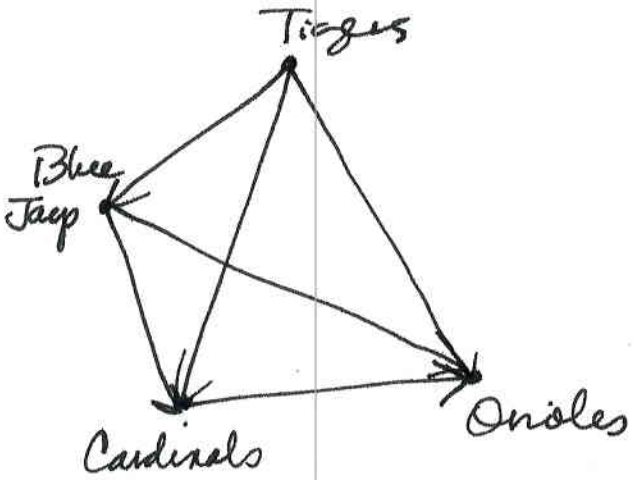
2366 Homework #7 Key

1.



Key:
$A \rightarrow B$ A defeats B

2a. directed, simple, connected (weakly)

b. undirected, simple, connected

c. undirected, pseudograph, multiple edges, connected

d. connected, undirected, simple

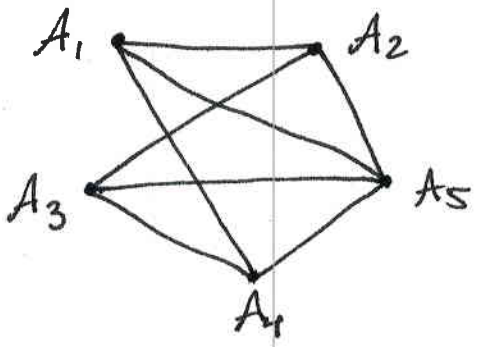
e. directed, simple, connected (weakly)

f. simple (weighted), undirected disconnected

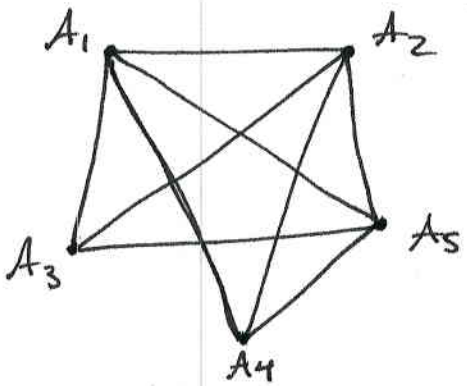
g. directed, multigraph, connected (weakly)

h. mixed, pseudograph, connected (weakly)

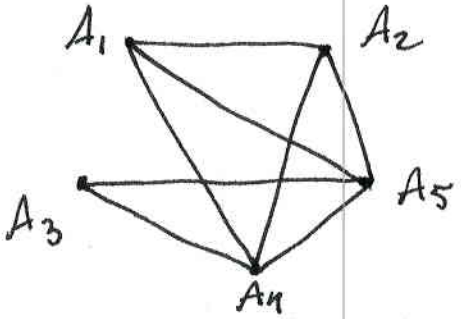
3. a.



b.



c.



4. Graph should be directed. For instance, a famous person is more likely to have their name be remembered than to remember names of non-famous people. And people have varying degrees of being good at remembering names. Multiple edges should only be allowed if directionality differs. No loops. One should assume everyone knows their own name, so this information would be unnecessary unless you were modeling amnesiacs.

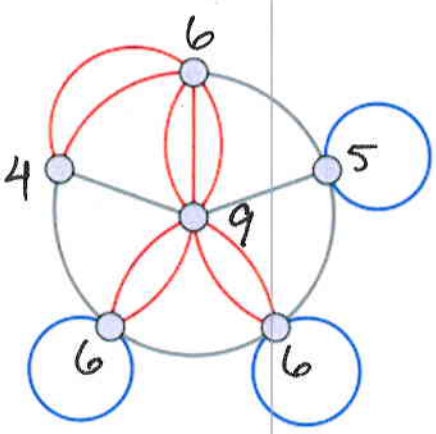
5. a. #1 $1^-, 0^+$ — out, + in

#2 $2^-, 1^+$

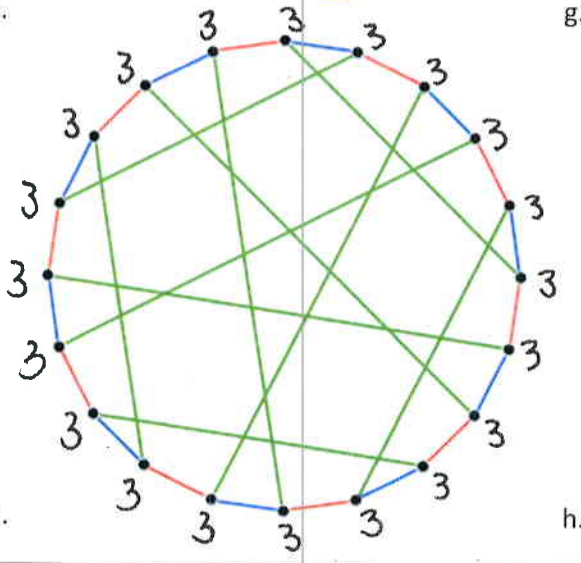
#3 $0^-, 2^+$ #4 $2^-, 1^+$ #5 $1^-, 1^+$ #6 $1^-, 2^+$

b. A. 3, B. 3, C. 1, D. 1, E. 2

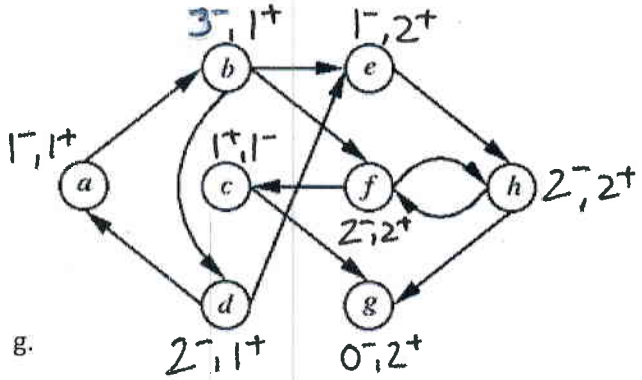
c.



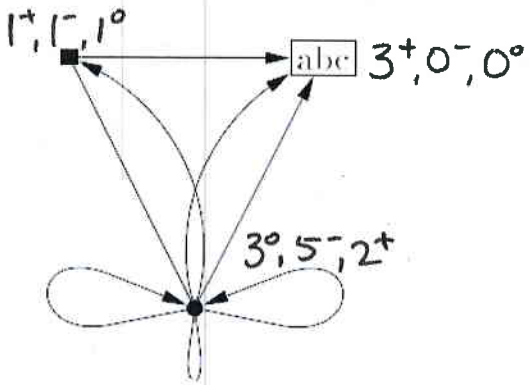
d.



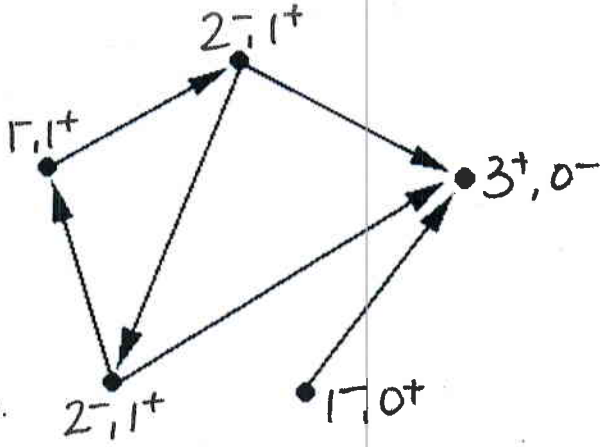
g.



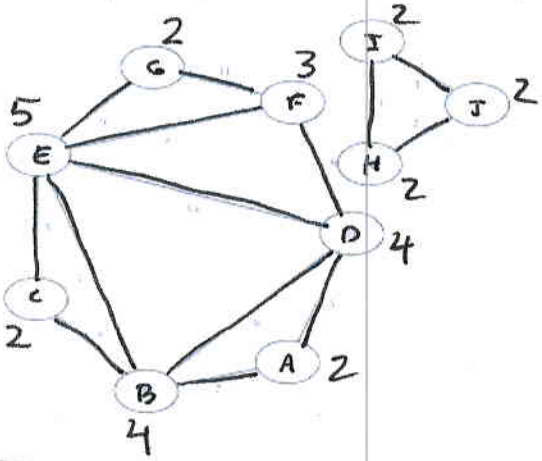
h.



e.



f.



6. a. $\frac{n(n-1)}{2}$

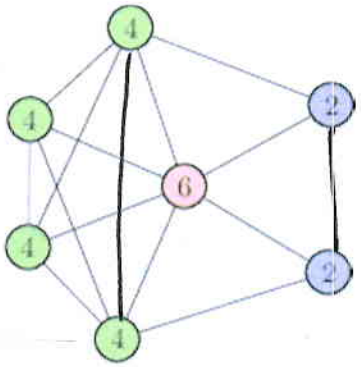
c. n

e. $2n$

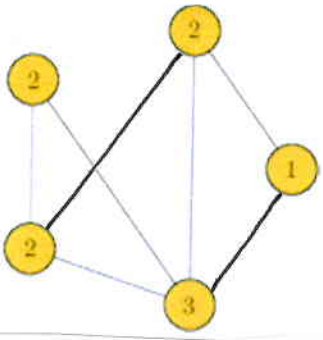
b. mn

d. 2^n

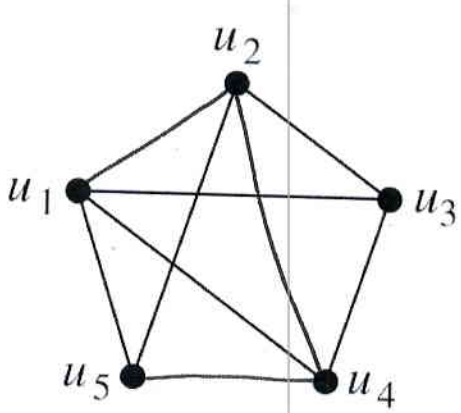
7a.



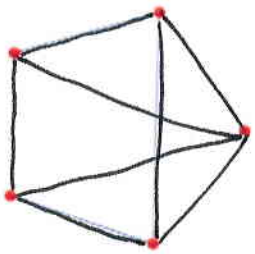
b.



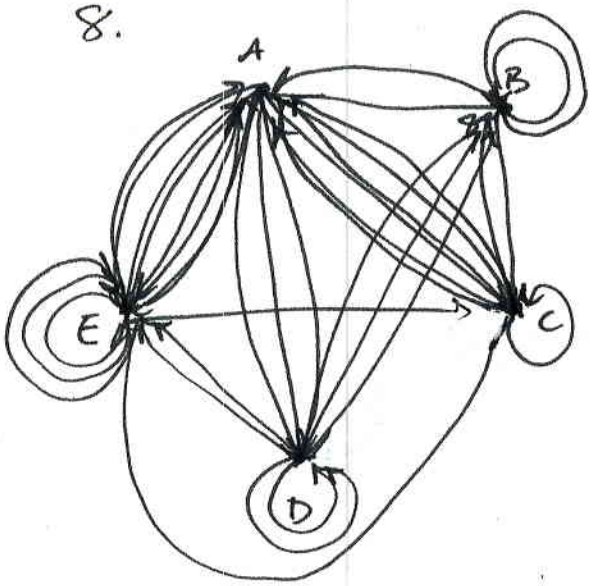
7c.



d.



8.



matrix is not symmetric so it must be directed

9. a. $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ b. $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$

10a. $\begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$ $\begin{array}{c} \begin{matrix} a & h & d & i & g & b & j & c \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$

They are isomophic

$a \Rightarrow 1$, $h \Rightarrow 2$, $d \Rightarrow 3$, $i \Rightarrow 4$, $g \Rightarrow 5$, $b \Rightarrow 6$, $j \Rightarrow 7$, $c \Rightarrow 8$

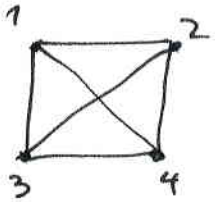
b. not isomophic first graph has vertices w/ degrees $\{2, 3, 3, 3, 3, 2\}$ & second has $\{2, 3, 3, 2, 2, 4\}$.

c. not isomophic. in first graph. there is no 5-circuit from v_1 to v_1 but there is in second graph.

d. not isomophic 4-circuit in 1st graph & 3-circuit in 2nd

e. not isomophic since degrees of vertices in 1st are $\{1, 1, 1, 3\}$ but second is $\{1, 2, 2, 1\}$.

11. K_4 $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} = A$



$A^2 = \begin{bmatrix} 3 & 2 & 2 & 2 \\ 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 2 \\ 2 & 2 & 2 & 3 \end{bmatrix}$ Therefore There are 2 paths of length 2 between any pair of non-identical vertices

11 cont'd

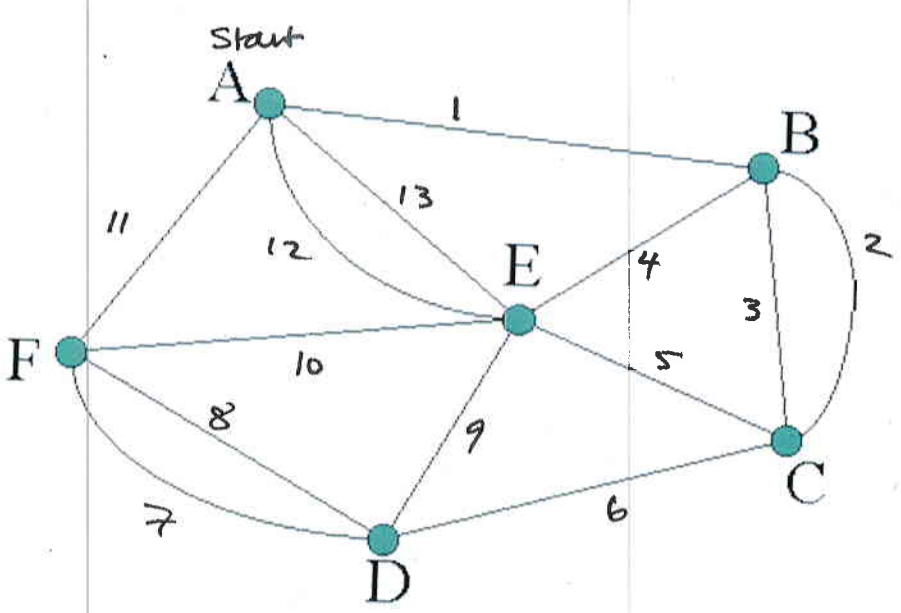
$A^3 = \begin{bmatrix} 6 & 7 & 7 & 7 \\ 7 & 6 & 7 & 7 \\ 7 & 7 & 6 & 7 \\ 7 & 7 & 7 & 6 \end{bmatrix}$ 7 different paths of length 3

$A^4 = \begin{bmatrix} 21 & 20 & 20 & 20 \\ 20 & 21 & 20 & 20 \\ 20 & 20 & 21 & 20 \\ 20 & 20 & 20 & 21 \end{bmatrix}$ 21 different paths of length 4

$A^5 = \begin{bmatrix} 60 & 61 & 61 & 61 \\ 61 & 60 & 61 & 61 \\ 61 & 61 & 60 & 61 \\ 61 & 61 & 61 & 60 \end{bmatrix}$ 61 different paths of length 5

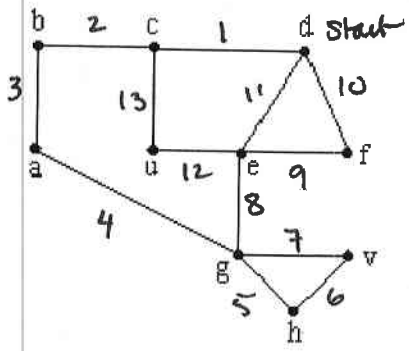
12. a. no Euler circuit a path; too many odd vertices

b. answers will vary



b.

c. Euler path, answers will vary



c.

12 cont'd

d. Euler path, answers will vary.

13a. no Hamilton path or circuit

each path misses one vertex.

b. there is a Hamilton path

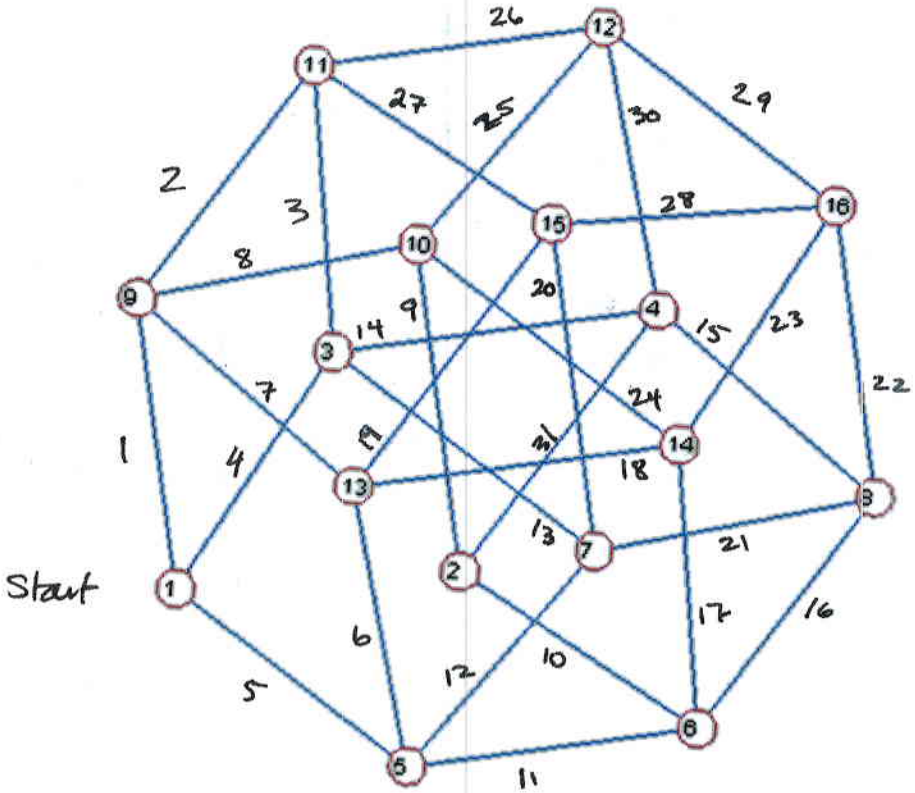
D A E B F C

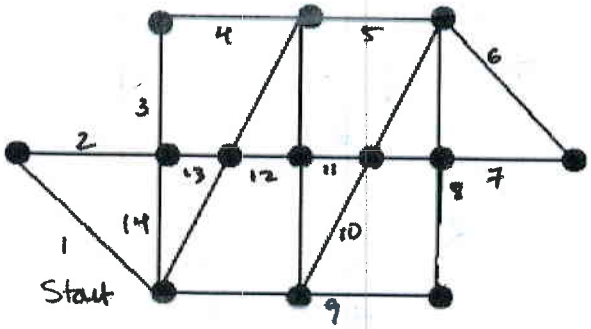
no circuit

c. is a Hamilton Circuit

1, 6, 4, 2, 3, 5, 1

d. is a Hamilton circuit





d.