

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. Find $f(2), f(3), f(4), f(5)$ if $f(0) = 3, f(1) = 1$, and f is defined recursively as $f(n+1) = f(n)^2 - f(n-1)^3$.

$$f(2) = 1^2 - 3^3 = 1 - 27 = -26$$

$$f(3) = (-26)^2 - 1^3 = 675$$

$$f(4) = (675)^2 - (-26)^3 = 473,201$$

$$f(5) = (473,201)^2 - (675)^3 \approx 2.236 \times 10^{11}$$

2. Define well-formed formulae of sets using the operators $\bar{}, \cup, \cap, -$.

$$\bar{A}, A \cup B, A \cap B, A - B$$

to combine w/ additional operators put parentheses around each compound set and then a single set combined w/ or 2 sets on either side of other operators compound or single sets

3. How many strings are there of 4 lowercase letters or fewer not counting the empty string?

$$26^1 + 26^2 + 26^3 + 26^4 = 475,254$$

4. How many strings of 8 alphanumeric characters are possible if upper and lowercase are different, and no letter can be repeated, and it can have only one vowel?

one vowel \Rightarrow 21 possible consonants + 10 #'s
 $\times 2 = 42$ upper & lower = 52 non-vowels
 5 vowels $\times 2 = 10$ vowels

7 non-vowels

$$10 \cdot P(52, 7) \cdot \binom{8}{1} \approx 5.39 \times 10^{13}$$

\uparrow vowel choices \uparrow other letters \uparrow ordering of vowel

5. How many positive integers not exceeding 100 are divisible by either 4 or 6?

$$\frac{100}{4} = 25$$

$$\frac{100}{6} = 16$$

$$25 + 16 - 8 = 33$$

both $\rightarrow \frac{100}{12} = 8$

6. What is the minimum number of students a university must have to ensure that there are at least 100 students from the same state?

$$99 \times 50 + 1 = 4951$$

7. Show that if there are 30 students in a class, there are at least two of them with the same first initial.

There are only 26 letters, so the 27th student is guaranteed to have the same first initial as one of the previous 26. So, 30 is more than that.