2415 Homework #1 Key

1a. $y_1 = e^t \quad y_1' = e^t \quad y_1'' = e^t \qquad y'' - y = e^t - e^t = 0$ ✓

$y_2 = \cosh t \quad y_2' = \sinh t \quad y_2'' = \cosh t \quad y'' - y = \cosh t - \cosh t = 0$ ✓

b. $y = 3t + t^2 \quad y' = 3 + 2t \qquad ty' - y = t(3+2t) - (3t+t^2)$
$= 3t + 2t^2 - 3t - t^2 = t^2$ ✓

c. $y_1 = \frac{1}{3}t \quad y_1' = \frac{1}{3} \quad y_1'' = 0 = y_1''' = y_1^{iv} \quad y^{iv} + 4y''' + 3y = 0 + 0 + 3(\frac{1}{3}t) = t$ ✓

$y_2 = \frac{1}{3}t + e^{-t} \quad y_2' = \frac{1}{3} - e^{-t} \quad y_2'' = e^{-t} \quad y_2''' = -e^{-t} \quad y_2^{iv} = e^{-t}$

$y^{iv} + 4y''' + 3y = e^{-t} - 4e^{-t} + 3e^{-t} + 3(1/3 t) = t$ ✓

d. $y = \cos t \ln \cos t + t \sin t \quad y' = -\sin t \ln \cos t + \cancel{\cos t} \cdot \frac{-\sin t}{\cancel{\cos t}} + \sin t + t\cos t$
$= -\sin t \ln \cos t + t\cos t$

$y'' = -\cos t \ln \cos t + \sin t \frac{\sin t}{\cos t} + \cos t - t\sin t$

$y'' + y = -\cancel{\cos t \ln \cos t} + \frac{\sin^2 t}{\cos t} + \frac{\cos^2 t}{\cos t} - \cancel{t\sin t} + \cancel{\cos t \ln \cos t} + \cancel{t\sin t}$

$= \frac{1}{\cos t} = \sec t$

e. $y = e^{t^2}\int_0^t e^{-s^2}\,ds + e^{t^2} \quad y' = 2te^{t^2}\int_0^t e^{-s^2}\,ds + 2te^{t^2} + \underbrace{e^{t^2} \cdot e^{-t^2}}_{=1}$

$y' - 2ty = 2te^{t^2}\int_0^t e^{-s^2}\,ds + 2te^{t^2} + 1 - 2t\left(e^{t^2}\int_0^t e^{-s^2}\,ds + e^{t^2}\right)$

$= \cancel{2te^{ts}\int_0^t e^{-s^2}\,ds} + \cancel{2te^{t^2}} + 1 - \cancel{2te^{t^2}\int_0^t e^{-s^2}\,ds} - \cancel{2te^{t^2}} = 1$ ✓

2.a. $y = e^{rt} \quad y' = re^{rt} \quad y'' = r^2e^{rt} \qquad r^2e^{rt} + re^{rt} - 6e^{rt} = 0$

$e^{rt}(r^2 + r - 6) = 0 \qquad (r+3)(r-2) = 0 \qquad r = -3, r = 2$

b. $y = t^r \quad y' = rt^{r-1}, \quad y'' = r(r-1)t^{r-2} \quad t^2 \cdot r(r-1)t^{r-2} + 4t \cdot rt^{r-1} + 2t^r = 0$

$t^r(r^2 - r + 4r + 2) = t^r(r^2 + 3r + 2) = 0 \quad (r+1)(r+2) = 0$

$r = -1, \ r = -2$

3a. $y' - 2y = 3e^t \quad p(t) = -2 \quad \mu = e^{\int -2dt} = e^{-2t}$

$e^{-2t}y' - 2e^{-2t}y = 3e^{-t}$

$\int (e^{-2t}y)' = \int 3e^{-t} \Rightarrow (e^{-2t}y = -3e^{-t} + C)e^{2t} \Rightarrow$

$y = -3e^t + Ce^{2t}$

b. $ty' + 2y = \sin t \Rightarrow y' + \frac{2}{t}y = \frac{\sin t}{t} \quad p(t) = \frac{2}{t} \quad \mu = e^{\int \frac{2}{t}dt} = e^{2\ln t} = t^2$

$t^2y' + 2ty = t\sin t \Rightarrow \int (t^2y)' = \int t\sin t$

±	u	dv
+	t	$\sin t$
−	1	$-\cos t$
	0	$-\sin t$

$t^2y = -t\cos t + \sin t + C$

$y = -\frac{\cos t}{t} + \frac{\sin t}{t^2} + \frac{C}{t^2}$

c. $ty' - y = t^2e^{-t} \Rightarrow y' - \frac{1}{t}y = te^{-t} \quad p(t) = -\frac{1}{t} \quad \mu = e^{\int -\frac{1}{t}dt} = e^{-\ln t} = \frac{1}{t}$

$\frac{1}{t}y' - \frac{1}{t^2}y = e^{-t} \Rightarrow \int \left(\frac{1}{t}y\right)' = \int e^{-t}dt \Rightarrow \frac{1}{t}y = -e^{-t} + C$

$y = -te^{-t} + Ct$

d. $y' - 2y = e^{2t} \quad y(0) = 2 \quad \mu = e^{\int -2dt} = e^{-2t}$

$e^{-2t}y' - 2e^{-2t}y = 1 \Rightarrow \int (e^{-2t}y)' = \int 1dt \Rightarrow e^{-2t}y = t + C$

$y = te^{2t} + Ce^{2t} \Rightarrow 2 = 0 + C$

$y = te^{2t} + 2e^{2t}$

e. $t^3y' + 4t^2y = e^{-t} \quad y(-1) = 0 \Rightarrow y' + \frac{4}{t}y = t^{-3}e^{-t}$

$\mu = e^{\int \frac{4}{t}dt} = e^{4\ln t} = t^4 \Rightarrow t^4y' + 4t^3y = te^{-t}$

	u	dv
+	t	e^{-t}
−	1	$-e^{-t}$
	0	e^{-t}

$\int (t^4y)' = \int te^{-t}dt \Rightarrow t^4y = -te^{-t} - e^{-t} + C$

$\Rightarrow y = -\frac{e^{-t}}{t^3} - \frac{e^{-t}}{t^4} + \frac{C}{t^4} \Rightarrow 0 = \frac{-e^1}{(-1)^3} - \frac{e^1}{(-1)^4} + \frac{C}{(-1)^4}$

3e cont'd

$\Rightarrow C=0 \qquad y = -\frac{e^{-t}}{t^3} - \frac{e^{-t}}{t^4}$

4. a. $\mu = e^{-2t}$

$y = e^{2t}\int e^{-2t}3e^{t}\,dt = e^{2t}\int 3e^{-t}\,dt = -3e^{2t}e^{-t} + Ce^{2t}$

$= -3e^{t} + Ce^{2t}$

b. $\mu = t^2 \qquad y = \frac{1}{t^2}\int t^2\frac{\sin t}{t}\,dt = \frac{1}{t^2}\int t\sin t\,dt =$

$\frac{1}{t^2}[-t\cos t + \sin t + C] = -\frac{\cos t}{t} + \frac{\sin t}{t^2} + \frac{C}{t^2}$

c. $\mu = \frac{1}{t} \qquad y = t\int \frac{1}{t}\cdot te^{-t}\,dt = t\int e^{-t}\,dt = t[-e^{-t} + C]$

$y = -te^{-t} + Ct$

d. $\mu = e^{-2t} \qquad y = e^{2t}\int e^{-2t}\cdot e^{2t}\,dt = e^{2t}[t + C] = te^{2t} + Ce^{2t}$

$2 = 0 + Ce^{0} \Rightarrow C=2$

$y = te^{2t} + 2e^{2t}$

e. $\mu = t^4 \qquad y = \frac{1}{t^4}\int t^4\frac{e^{-t}}{t^3}\,dt = \frac{1}{t^4}\int te^{-t}\,dt = \frac{1}{t^4}[-te^{-t} - e^{-t} + C]$

$y = -t^{-3}e^{-t} - t^{-4}e^{-t} + Ct^{-4}$

$0 = -(-1)^{-3}e^{1} - (-1)^{-4}e^{1} + C(-1)^{-4} = \cancel{e} - \cancel{e} + C \Rightarrow C=0$

$y = -e^{-t}t^{-3} - e^{-t}\cdot t^{-4}$