Math 2415, Quiz #10, Summer 2014 Name KEY

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. Find the eigenvalues and eigenfunctions for the differential equation $y'' + \lambda y = 0, y(0) = 0,$ $y(L) = 0$. Be sure to check all possible cases.

$r^2+\lambda=0$

$r^2=-\lambda$

$r=\pm\sqrt{-\lambda}$

if $\lambda=0$

then $y=At+B$

since $y''=0$

$y(0)=0=B \Rightarrow B=0$

$y(L)=0=A(L) \Rightarrow A=0$

$y=0$

if $\lambda=-\mu^2$

$r=\pm\sqrt{\mu^2}=\pm\mu$

$y=Ae^{\mu x}+Be^{-\mu x}$

or $A\cosh\mu x + B\sinh\mu x$

$y(0)=A+B=0$

$y(L)=Ae^{\mu L}+Be^{-\mu L}=0$

$A=-B$

$Ae^{\mu L}=-Be^{-\mu L})e^{\mu L}$

$Ae^{2\mu L}=-B$

$Ae^{2\mu L}=A \Rightarrow A(e^{2\mu L}=1)$

$e^{2\mu L}=1$

$2\mu L=0$

no solution if $\mu\neq 0$ & $L\neq 0$

if $\lambda=\mu^2$

$r=\pm\mu i$

$y=A\cos\mu x+B\sin\mu x$

$y(0)=A=0$

$y(L)=B\sin(\mu L)=0$

if $B\neq 0$ $\mu L=n\pi$

$\Rightarrow \mu=\frac{n\pi}{L}$

$y=B\sin\left(\frac{n\pi x}{L}\right)$

not unique

2. Find the Fourier series for the given function $f(x) = x, -1 \le x < 1, f(x+2) = f(x)$.

$a_0=\int_{-1}^{1} x\,dx = \left.\frac{x^2}{2}\right|_{-1}^{1}=\frac{1}{2}-\frac{1}{2}=0$

$a_n=\int_{-1}^{1} x\cos(n\pi x)\,dx = \cancel{\frac{x}{n\pi}\sin(n\pi x)}^{0}+\left.\frac{1}{n^2\pi^2}\cos(n\pi x)\right|_{-1}^{1}=\frac{1}{n^2\pi^2}\left[\cos(n\pi)-\cos(n\pi)\right]=0$

this makes sense since x is odd.

$b_n=\int_{-1}^{1} x\sin(n\pi x)\,dx = -\frac{x}{n\pi}\cos(n\pi x)+\left.\cancel{\frac{1}{n^2\pi^2}\sin(n\pi x)}^{0}\right|_{-1}^{1} = \frac{-1}{n\pi}\cos(n\pi)+\frac{-1}{n\pi}\cos(n\pi)$

$=\frac{-2}{n\pi}(-1)^n$

$$f(x)=\sum_{n=1}^{\infty}\frac{-2(-1)^n}{n\pi}\sin(n\pi x)$$